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CLASSIFICATION OF δ-INVARIANT AMALGAMATION CLASSES

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§1. Introduction. Hrushovski's generalization of the Fraisse construction has provided a rich source of examples in model theory, model theoretic algebra and random graph theory. The construction assigns to a dimension function δ and a class $\mathbf{K}$ of finite (finitely generated) models a countable 'generic' structure. We investigate here some of the simplest possible cases of this construction. The class $\mathbf{K}$ will be a class of finite graphs; the dimension, $\delta(A)$, of a finite graph A will be the cardinality of A minus the number of edges of A. Finally and significantly we restrict to classes which are δ-invariant. A class of finite graphs is δ-invariant if membership of a graph in the class is determined (as specified below) by the dimension and cardinality of the graph, and dimension and cardinality of all its subgraphs. Note that a generic graph constructed as in Hrushovski's example of a new strongly minimal set does not arise from a δ-invariant class.

We show there are countably many δ-invariant (strong) amalgamation classes of finite graphs which are closed under subgraph and describe the countable generic models for these classes. This analysis provides ω-stable generic graphs with an array of saturation and model completeness properties which belies the similarity of their construction. In particular, we answer a question of Baizhanov (unpublished) and Baldwin [5] and show that this construction can yield an ω-stable generic which is not saturated. Further, we exhibit some ω-stable generic graphs that are not model complete. Most of the definitions and notation are carried over from [1] or from Baldwin and Shi [4]. The existence of this variety of examples for graphs with the simplest choice of dimension function shows the diversity of even the class of theories with trivial forking.

§2. Preliminaries. For ease of discussion, we introduce some notation. A graph is a structure with one binary relation which is symmetric and irreflexive. L_n denotes the 'line' of length n, a graph on $\{a_0, \dots, a_{n-1}\}$ in which only the vertices a_i, a_{i+1}, $i < n-2$ are connected by edges. We say L_n is a *chain* of length $n-1$. The graph obtained from L_n by connecting the vertices a_0 and a_n is called the *cycle*, C_n, of length n. If the finite graphs B and C intersect in A and there are no edges from $B - A$ to $C - A$, we call the graph $B \cup C$ the *free amalgamation* of B and C over A and denote it by $B \otimes_A C$. The direct sum of two graphs is their free amalgamation

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over the empty set. We denote the direct sum of n copies of a graph G by G^n. A graph A is called *connected* if it cannot be written as a direct sum of two proper nonempty subgraphs. A connected graph with no cycles is called a *tree*. A *full tree* is a countable tree in which every vertex has an infinite number of vertices attached to it. We denote a full tree by T. A *discrete graph* on m points, that is, a collection of points with no edges, is denoted by D_m. An n-cycle is a *hairy cycle* if it has a full tree attached to every vertex; we denote such a graph by H_n. A *furry n-cycle* is a graph with $k > n$ vertices which contains a unique n-cycle. A *furry cycle* is a furry n-cycle for some n. For emphasis we may write *bald n-cycle* for C_n. A class (which is closed under subgraph) is said to *omit* a graph B, if B is not a member of the class. Similarly, a class *allows* or *admits* a graph B if it is a member of the class. Let $y(B)$ denote $\langle |B|, e(B)\rangle$ where $e(B)$ is the number of edges in the graph.

DEFINITION 2.1. Let **K** be a class of finite graphs closed under isomorphisms. **K** is *δ-invariant* if for any finite A, $A \in$ **K** if and only if for every subgraph B of A, $\delta(B) \geq 0$ and there is a $B' \in$ **K** such that $y(B) = y(B')$.

Note that a δ-invariant class **K** is closed under subgraphs. Observe that the strongly minimal examples of [8] are *not* δ-invariant. The above definitions yield: $\delta(C_n) = 0$ and $\delta(L_n) = 1$, for any $n < \omega$. One can check by induction on cardinality that a connected graph which contains no cycle has dimension 1 and a connected graph which contains a single cycle has dimension 0. Furthermore,

PROPOSITION 2.2. *Let $A \in$ **K** be a connected graph. $\delta(A) = 0$ if and only if A contains exactly one cycle.*

PROOF. Suppose A contains two cycles C_n and C_m. If they overlap it is easy to check that $\delta(C_n C_m) < 0$. If they are disjoint, since A is connected, there is a chain K joining them; note that $\delta(C_n K C_m) = -1$. ⊣

DEFINITION 2.3.

1. We say A is *strong* in B and write $A \leq B$, if $\delta(A) \leq \delta(A')$ for all A' with $A \subseteq A' \subseteq B$.
2. The class **K** has the strong or $\leq$-amalgamation property if every pair of strong extensions B, C of a graph $A \in$ **K** have a common strong extension in **K** (by maps which commute in the usual manner).

An easy calculation yields:

PROPOSITION 2.4. *If $A \leq B$ then for any $C_m \subseteq B$ either $C_m \subseteq A$ or $C_m \subseteq B \setminus A$.*

In the usual way if a class has the amalgamation property (we often omit the $\leq$) there is a unique countable generic model in the following sense.

DEFINITION 2.5. The model M is (**K**, $\leq$)*-homogeneous-universal* or (K, $\leq$)*-generic* if it satisfies following conditions:

1. If $A \leq M$, $A \leq B \in$ **K**, then there exists $B' \leq M$ such that $B \cong_A B'$,
2. M is the union of an increasing sequence $\{A_n : A_n \in$ **K**$\}$ of finite strong substructures.

§3. δ-invariant amalgamation classes. In this section we characterize the δ-invariant amalgamation classes. By Proposition 2.2 any connected graph in $\mathbf{K}$ can have at most one cyclic subgraph. As Theorem 3.7 points out, if $K_3 \in \mathbf{K}$ then by $\delta-$invariance any tree is in $\mathbf{K}$. Thus the δ-invariant classes which admit K_3 are distinguished only by which graphs of dimension 0 they admit. On the other hand, if K_3 is not allowed, then the class is much simpler; only direct sums of points, 3-cycles, and K_2s are allowed. Before establishing these facts, we define the following classes:

DEFINITION 3.1.

1. $\mathbf{K}_0$ is the class of graphs which omit K_2.
2. For $m \leq \omega$, $\mathbf{K}_{1,m}$ is the class of finite graphs which omit K_3 and C_3^k for all $k > m$.
3. $\mathbf{K}_{2,I}$ is the class of finite graphs (with hereditarily nonnegative dimension) which omit all finite graphs A with $\delta(A) = 0$ and $|A| \notin I$ where the set $I \subset \omega$ has the following properties:
   (a) $m, n \in I$ and $m \neq n \Longrightarrow m + n \in I$
   (b) $n, n+1 \in I \Longrightarrow \forall m \geq 0\ (n + m \in I)$.

THEOREM 3.2. *There are exactly three possible types of amalgamation classes which are δ-invariant. These classes are:*

1. *the class* $\mathbf{K}_0$,
2. *the classes* $\mathbf{K}_{1,m}$,
3. *the classes* $\mathbf{K}_{2,I}$.

This theorem will follow from the following lemmas. The first is obvious

LEMMA 3.3. *The class* $\mathbf{K}_0$ *contains exactly those finite graphs which consist of vertices with no edges; it is δ-invariant and has the amalgamation property.*

We can characterize all $\leq$-amalgamation classes which are closed under substructures and omit K_3. They are all easily seen to be δ-invariant.

LEMMA 3.4.

1. *A graph is a member of the class* $\mathbf{K}_{1,m}$ *if and only if it has the following form:* $D_l \oplus K_2^n \oplus C_3^k$, *where* $l, n < \omega$ *and* $k \leq m$.
2. *The members of the set:* $\{\mathbf{K}_{1,m} : m \leq \omega\}$ *are all the possible classes, closed under substructures, which omit* K_3 *but not* K_2, *and have the* $\leq$*-amalgamation property.*

PROOF. Since every finite graph is a direct sum of connected graphs and every connected graph except D_1, K_2 and C_3 embeds K_3 part one of the lemma is obvious.

By amalgamation over the empty set it is easily seen that any amalgamation class, closed under substructures, which contains K_2 contains $D_l \oplus K_2^n$ for all $l, m < \omega$. Thus the amalgamation classes which omit K_3 are distinguished only by which graphs C_3^k they admit.

Finally, for each m, we check that $\mathbf{K}_{1,m}$ is closed under amalgamation. Let B, $C \in \mathbf{K}_{1,m}$, $A = B \cap C$, $A \leq B$, $A \leq C$ and suppose E is the free amalgamation of B and C over A. From part 1 of this lemma , we know B and C can be written in the form $D_l \oplus K_2^n \oplus C_3^k$, and $D_{l'} \oplus K_2^{n'} \oplus C_3^{k'}$, respectively, and without loss of generality, $k' \geq k$. Since $A \leq B$ and $A \leq C$, no element of $(B \cup C) \setminus A$ can be connected to more than one element of A. Moreover each cycle embedded in E is embedded in one of A, $B \setminus A$, $C \setminus A$. Since $A \in \mathbf{K}_{1,m}$, $A \simeq D_{l''} \oplus K_2^{n''} \oplus C_3^{k''}$. So $E \simeq D_{l'''} \oplus K_2^{n'''} \oplus C_3^{k'''} \oplus K_3^{j}$, where $l''' = l + l' - l'' - j$, $k''' = k + k' - k''$ and $n''' = n + n' - n'' - 2j$. If E contained a copy of K_4 one of B, C would contain a copy of K_3. To find an amalgam of B and C over A which is in $\mathbf{K}_{1,m}$, identify the ends of any K_3 in E and identify each copy of C_3 in $B \setminus A$ with a copy of C_3 in $C \setminus A$. ⊣

Finally we look at the most complicated of the δ-invariant classes. In order to get some feel for what these classes look like, consider the following example, which uses Lemma 3.7 to describe the class $\mathbf{K}_{2,I}$, for a particular I.

Example 3.5. Let $I = \{3, 7, 10, 13, 16, 17, 18, \dots\}$; first notice that I satisfies the properties in Definition 3.1. Now, C_n is a subgraph of a graph in $\mathbf{K}_{2,I}$ if and only if $n \in I$. In particular C_3, C_7, C_{10}, $C_{13} \in \mathbf{K}_{2,I}$ and there are no furry n-cycles in the class for those n. Since 6, 14 $\notin I$, we can see that C_3, and C_7 occur uniquely in any graph which is a member of $\mathbf{K}_{2,I}$. But there are direct sums of any finite number of C_ns, where $n \in I$, and $n > 7$, in graphs belonging to this class. Finally, for any $n \geq 16$, $\mathbf{K}_{2,I}$ admits both furry and bald n-cycles.

Notation 3.6. We say I contains an *interval* if there exists an $n \in I$ such that for $m \geq n$, m is in I. We denote the least such n as i_0 and denote the interval by $\bar{I}$. The set of elements $n \in I$ such that $2n \notin I$ will be denoted by I'.

Lemma 3.7.

1. *Each member of the class* $\mathbf{K}_{2,I}$ *is a direct sum of a finite number of finite graphs of the following types*:
   (a) *a tree*
   (b) C_n *if* $n \in I$ *and* $2n \in I$,
   (c) *furry* m*-cycles, with any finite cardinality* $> m$, *for all* $m \geq i_0$, *if* I *contains an interval,*

   and at most one C_n *if* $n \in I$ *and* $2n \notin I$.
2. *The* $\mathbf{K}_{2,I}$ *for* $I \subset \omega$ *which have the properties in Definition* 3.1 *are all the possible* δ*-invariant amalgamation classes, which admit* K_3.

Proof. For part 1, we must show that the finite graphs which omit those A with $\delta(A) = 0$ and $|A| \notin I$ have the form described. Using the definition of I, one can see fairly easily that direct sums of the graphs described are in $\mathbf{K}_{2,I}$. Any finite graph B with $\delta(B) \geq 0$ can be decomposed as a direct sum of trees, bald cycles, and furry cycles. For the converse we must show that the components of a $B \in \mathbf{K}_{2,I}$ satisfy the indicated conditions. In order for a furry m-cycle F to be a member of $\mathbf{K}_{2,I}$, m and $m + 1$ must be in I. By the second part of the definition of I, this implies that $m \geq i_0$. The other two conditions are easier.

For part 2, we first show each class $\mathbf{K}_{2,I}$, has the amalgamation property. Let $C, B \in \mathbf{K}_{2,I}$, $A = B \cap C$, $A \leq B$, and $A \leq C$. Lemma 3.8 (below) implies that $B \otimes_A C \in \mathbf{K}_{2,I}$ if and only if $B \otimes_A C$ contains only one copy of a bald cycle C_m, where $2m \notin I$ and if a furry n-cycle F is contained in $B \otimes_A C$ then $|F| \in I$. To verify these conditions, suppose first there is more than one bald cycle C_m in the free amalgam. Define an embedding $f' \colon B \setminus A \to C \setminus A$ which identifies the unique bald cycle in $B \setminus A$ with the corresponding bald cycle in $C \setminus A$. Let $f = f' \cup \mathrm{id}_{B \setminus B'}$. Note that $(B \setminus B') \otimes_A C$ equals $((B \setminus B') \cup f(B')) \otimes_A (C \setminus f(B'))$. But the first factor of the second expression is just $f(B)$. Thus, $f(B) \leq (B \setminus B') \otimes_A C$ and $(B \setminus B') \otimes_A C \in \mathbf{K}_{2,I}$ as required.

On the other hand, if a furry n-cycle F is contained in $B \otimes_A C$ then a furry n-cycle F' with $|F'| = n + 1$ must be embedded in B or C and so $n \in \bar{I}$ and $|F| \in I$.

It remains to show that a minimal δ-invariant $\leq$-amalgamation class $\mathbf{K}$, which admits K_3 has the form $\mathbf{K}_{2,I}$ for some I satisfying the conditions of Definition 3.1 part 3. We claim first that such $\mathbf{K}$ admit all finite trees. It is easy to see that if $K_3 \in \mathbf{K}$ then $L_n \in \mathbf{K}$ for all n. But, by δ-invariance, if every L_n is in $\mathbf{K}$, any tree is in $\mathbf{K}$.

Let $J = J_{\mathbf{K}} = \{n : \mathbf{K} \text{ admits } C_n\}$. We now show J satisfies the conditions described in Definition 3.1 part 3. First, if $C_n, C_m \in \mathbf{K}$ and $n \neq m$, amalgamation over the empty set shows that $C_m \oplus C_n \in \mathbf{K}$, giving us Definition 3.1 part 3a. Moreover, if $C_n, C_{n+1} \in \mathbf{K}$, then for all $m \geq n$, $C_m \in \mathbf{K}$. For this, consider the following amalgamation. Form B and C by attaching a point to consecutive vertices of C_n; by δ-invariance, B and C are in $\mathbf{K}$. Now, the amalgamation of B and C over C_n cannot identify the new points as the resulting graph would have negative dimension. So the amalgam is a graph with cardinality $n + 2$, and dimension 0; by δ-invariance, we deduce C_{n+2} is in $\mathbf{K}$. This gives Definition 3.1 part 3b.

Finally $\mathbf{K}$ is $\mathbf{K}_{2,J}$. We have both $\mathbf{K}$ and $\mathbf{K}_{2,J}$ contain all finite direct sums of connected graphs of positive dimension. $\mathbf{K}$ and $\mathbf{K}_{2,J}$ contain the same cycles and $\mathbf{K}_{2,J}$ is the largest class of finite graphs containing exactly those cycles so $\mathbf{K} \subseteq \mathbf{K}_{2,J}$. Now, if B in $\mathbf{K}_{2,J}$ is a direct sum of cycles and furry cycles, we show first that each (furry) cycle is in $\mathbf{K}$ and then by induction on the number of summands that $B \in \mathbf{K}$. For bald cycles the first claim is evident by the definition of J; for furry k-cycles F it follows from the observation that if $F \in \mathbf{K}_{2,J}$ then $k \in \bar{J}$. For the induction step note that any subgraph of B is a direct sum of trees and a graph of the desired sort with fewer summands. Since $\mathbf{K}$ has amalgamation this direct sum B is in $\mathbf{K}$ and we finish. ⊣

LEMMA 3.8. *Let $A \leq B$, $A \subseteq C$, then $B \otimes_A C \in \mathbf{K}_{2,I}$ if and only if the following two statements hold*:

1. *if $C_k^2 \subseteq B \otimes_A C$ then $2k \in I$*
2. *if $E \subseteq B \otimes_A C$ is a furry cycle then $|E| \in I$.*

PROOF. Clearly, if the free amalgam is in the class each of the two statements holds. For the converse, let E be a minimal graph in $B \otimes_A C$ such that $E \notin \mathbf{K}_{2,I}$.

Suppose E is not connected, say $E = E_1 \oplus E_2$. Then $E_1, E_2 \in \mathbf{K}_{2,I}$ (by minimality of E), but $E_1 \oplus E_2 \notin \mathbf{K}_{2,I}$. It follows from Lemma 3.7 part 1, that for some k, $E_1 = E_2 = C_k$ and $2k \notin I$.

Suppose E is connected. Let $C_0 = E \cap (C \setminus A)$, $B_0 = E \cap (B \setminus A)$, and $A_0 = E \cap A$. Since $C_0, A_0, B_0 \in \mathbf{K}_{2,I}$ and $A_0 \leq B_0$, $\delta(E) = \delta(C_0) + (\delta(B_0) - \delta(A_0)) \geq 0$. Since E is not in $\mathbf{K}_{2,I}$, $\delta(E) = 0$ and thus E contains exactly one cycle. Furthermore, since B_0 and C_0 are not empty (otherwise E would be a subgraph of a member of the class) and $A \leq B$, E is a furry cycle. By hypothesis $|E| \in I$, and by the minimality of E, all of E's subgraphs are in $\mathbf{K}_{2,I}$, thus $E \in \mathbf{K}_{2,I}$. This contradiction yields the result. ⊣

With a little number theory, we establish the following result.

COROLLARY 3.9. *There are only countably many δ-invariant amalgamation classes of graphs (with $\delta(A) = |A| - e(A)$).*

PROOF. Let I be any infinite set of natural numbers such that $m \neq n \in I$ implies $m + n \in I$. Let r be the greatest common divisor of the elements of I. (Note $1 \leq r \leq \min(I)$.) Then there exist $p > 0$ and $m_0, \ldots, m_p,\ a_0, \ldots, a_p$ with the $m_i \in I$ and nonzero integers a_i such that $\Sigma_{i \leq p} a_i m_i = r$. Let M be the minimum of the m_i. Let $a_i' = M|a_i|$ for $a_i < 0$ and $a_i' = a_i$ otherwise. Since any linear combination of the m_i with strictly positive coefficients is in I, $N = \Sigma_{i \leq p} a_i' m_i \in I$; we show for all u, $N + ur \in I$ and these are exactly the members of I which are at least N. First let $u < M$. We see $N + ur = N + u(\Sigma_{i \leq p} a_i m_i) \in I$, since for all i, $a_i' > u a_i$ if $u < M$ (so again the coefficients of the m_i are positive). Now, $N + Mr = (N + M) + \cdots + M$ which is a sum of distinct members of I; we finish by induction. Conversely, if $N \leq w \in I$ then $w = N + ur$ for some u. For, if N, $N + k \in I$, then r divides their difference k (since r is the greatest common divisor of I). Thus I is determined by r, N, and a finite set of elements less than N; there are only countably many such sets of natural numbers. ⊣

§4. Generics and ω-saturation. We now describe up to isomorphism the unique generic model of each of the above defined classes. We outline a proof for the most complicated case; the simpler cases can be proved in a similar manner. Recall Notation 3.6.

THEOREM 4.1. *The unique generic up to isomorphism of the class*

- $\mathbf{K}_0$ *is*: D_ω,
- $\mathbf{K}_{1.m}$ *is*: $K_2^\omega \oplus C_3^m$,
- $\mathbf{K}_{1.\omega}$ *is*: $K_2^\omega \oplus C_3^\omega$,
- $\mathbf{K}_{2.I}$ *is*: $T^\omega \oplus \sum_{k \in I \setminus (I' \cup \bar{I})} C_k^\omega \oplus \sum_{k \in \bar{I}} H_k^\omega \oplus \sum_{i \in I'} C_i$.

PROOF. Doing the last case, we show that a model M of the specified form is $(\mathbf{K}_{2.I}, \leq)$-homogeneous and $(\mathbf{K}_{2.I}, \leq)$-universal. The uniqueness of generic models yields M is indeed the generic for $\mathbf{K}_{2.I}$. Clearly M is $(\mathbf{K}_{2.I}, \leq)$-universal. For homogeneity, We must show that a local isomorphism between finite closed sets extends to an automorphism of M. The presentation of M as a direct sum allows us to restrict to finite connected strong subgraphs. But if A and A' are isomorphic connected strong subgraphs of M this isomorphism extends to the components they generate. For example, if C is a finite connected strong submodel of a hairy cycle H, then C must be a cycle, or a furry cycle. If C is a bald cycle C_n, it must be sent to another copy of C_n in M. But all copies of C_n are in copies of H_n and the isomorphism of C to C' extends to an automorphism of M taking H_n to H_n'. The other cases are similar. ⊣

When I is infinite but does not contain an interval, the generic model of $\mathbf{K}_{2.I}$ contains arbitrarily large bald cycles. Thus, the type of an element generating an infinite chain Z is consistent but not realized. We exhibit the saturated model of this theory in the proof of Theorem 4.4.

For each δ-invariant amalgamation class, the following table lists several important properties of the theory of the generic. The columns headed M. C. and Gen.

Sat. indicate whether the theory is model complete or the generic is saturated. Most of these properties can be established by routine analysis of the generic or saturated model.

	ω-stable	$\aleph_0$-cat.	$\aleph_1$-cat.	M. C.	Gen. Sat.
$\mathbf{K}_0$	Yes	Yes	Yes	Yes	Yes
$\mathbf{K}_{1.m}$	Yes	Yes	Yes	Yes	Yes
$\mathbf{K}_{1.\omega}$	Yes	Yes	No	No	Yes
$\mathbf{K}_{2.I}$ $\bar{I} = 0$ and $\lvert I\rvert = \omega$	Yes	No	No	No	No
$\mathbf{K}_{2.I}$ $\bar{I} = 0$ and $\lvert I\rvert \leq 1$	Yes	No	No	No	Yes
$\mathbf{K}_{2.I}$ $\bar{I} \neq 0$	Yes	No	No	No	Yes

PROPOSITION 4.2. $\mathrm{Th}(M, \mathbf{K}_{2.I})$ *is not model complete.*

PROOF. Let A be the generic of $\mathrm{Th}(M, \mathbf{K}_{2.I})$. Let $T \subset A$ be a full tree. Choose any vertex a in T with two neighbors, say b and c. Construct B, a model of $\mathrm{Th}(M, \mathbf{K}_{2.I})$, by removing a from A. Consider the formula σ: $\exists x\ (e(x,b) \wedge e(x,c))$. Now we notice, if there was a path from b to c, not through a, then T would have a cycle. Therefore $B \subset A$ and $A \models \sigma$ but $B \not\models \sigma$. ⊣

By embedding full trees in hairy cycles one can see that the class of models of $\mathrm{Th}(M, \mathbf{K}_{2.I})$ is not closed under unions of chains when $\bar{I} \neq \emptyset$. It is even easier to see $\mathrm{Th}(M, \mathbf{K}_{1.\omega})$ is not closed under unions of chains. So neither theory is Π_2-axiomatizable. Baldwin and Kueker noted (with different notation) [2] that $\mathbf{K}_{2.\emptyset}$ is Π_2-axiomatizable and not model complete. These results contrast with Holland's proof that the strongly minimal generic structures are model complete [7]. The following generalization of model complete was defined in [3].

DEFINITION 4.3. A theory T is said to be *nearly model complete* if every formula is equivalent in T to a Boolean combination of Σ_1-formulas.

THEOREM 4.4. *Let* $\mathbf{K}$ *be a* δ*-invariant amalgamation class.* $\mathrm{Th}(M, \mathbf{K})$ *is nearly model complete.*

PROOF. This result is obvious when the generic is ω-saturated, since $\Sigma_1 \cup \Pi_1$-types determine types in generic graphs. For $\mathbf{K}_{2.I}$ where I is infinite but does not contain an interval, we obtain the result by noticing that $\Sigma_1 \cup \Pi_1$-types determine types in the saturated model of this theory:

$$T^{\omega} \oplus \sum_{k \in I \setminus I'} C_k^{\omega} \oplus \sum_{i \in I'} C_i \oplus Z^{\omega}. \qquad ⊣$$

This paper was devoted to a very special case: δ-invariant amalgamation classes of symmetric, irreflexive binary relations. One possible extension is to consider wider classes of structures, e.g., arbitrary binary relations or ternary relations. Another is to consider strong amalgamation classes without the restriction of δ-invariance. This would of course require a different notion of what constitutes a classification since without δ-invariance there are $2^{\aleph_0}$ such classes even for the simple situation considered here.

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