

Scattering by a blade on a metallic plane*

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ABSTRACT

The scattering from a metal plane with a ridge is considered for the cases of plane wave and line source illumination and both polarizations. Exact analytical results are expressed using series that contain products of radial and angular Mathieu functions. The exact analytical results are computed and compared with high-frequency approximations and with measurements.

KEY WORDS

Electromagnetic scattering, High-frequency scattering, Mathieu functions

1 Introduction

The diffraction of electromagnetic waves by an infinitely long metallic strip, or blade, of finite height connected to a metallic ground plane and perpendicular to it is considered. The primary source is a plane wave propagating in a direction perpendicular to the edge of the strip and polarized with its electric field either parallel (E polarization) or perpendicular (H polarization) to the edge, or an electric or magnetic line source parallel to the edge. The resulting two-dimensional boundary-value problem is solved exactly in terms of infinite series of products of radial and angular Mathieu functions. The total field is written as the sum of three terms: the incident field, the field that would be reflected by the metallic ground plane in the absence of the blade, and the scattered field due to the presence of the blade; this last contribution obeys the two-dimensional radiation condition and, for plane wave incidence, leads to an exact formula for the radar cross section of the blade. The geometry of the problem is presented in section 2, and the exact results for plane wave and line source incidence are reported in sections 3 and 4, respectively. A high-frequency

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solution is proposed; it is developed in detail for plane wave incidence. It consists of considering the edge of the blade as the edge of a half-plane on which two fields are incident: the primary wave, and its image in the ground plane. Each of these two waves is diffracted by the edge of the blade, and the diffracted field reaches the observation point both directly and after reflection from the ground plane. Thus, four separate diffraction mechanisms contribute to the scattered field at the observation point. For plane wave incidence, each mechanism consists of the sum of two terms, each the product of a Fresnel integral and an exponential. Since the blade is of finite height, some reflected-field terms must be subtracted, depending on the position of the observation point. This formulation takes into account the dihedral effect of the blade/ground-plane corner, but does not account for the multiple interactions between the blade's edge and the blade/ground-plane corner; thus, it is expected to become a better approximation as the ratio of blade height to wavelength increases. This high-frequency result is simplified for the far backscattered field and compared with the exact solution in section 5, whereas numerical results from the exact solutions are compared to UTD results in section 6, for line source incidence. In the case of line source incidence, some bistatic results computed from the exact solutions for either polarization are also compared to measurements performed in an anechoic chamber, in section 7. In all cases, previously known results are not repeated, but may be easily found in the bibliography. In particular, a list of references on scattering by a strip may be found in [1]. The present work extends results presented by the authors in [2] and [3]. The frequency-domain analysis is conducted with a time-dependence factor $\exp(j\omega t)$, which is omitted throughout.

2 Geometry of the problem

The cross section of the two-dimensional problem is shown in Fig. 1. The axis $x = 0$ is a PEC plane and the blade OA is also a PEC. The points A and B are the foci of an elliptic coordinate system and their distance is the focal distance d . The relationship between elliptic cylinder and rectangular cartesian coordinates is

$$x = \frac{d}{2} \cosh u \cos v \quad (1)$$

$$y = \frac{d}{2} \sinh u \sin v \quad (2)$$

$$z = z \quad (3)$$

where

$$0 \leq u < \infty \quad 0 \leq v \leq 2\pi. \quad (4)$$

It is also convenient to introduce the variables (ξ, η, z) defined by

$$\xi = \cosh u \quad \eta = \cos v \quad (5)$$

where obviously $\xi \geq 1$ and $-1 \leq \eta \leq 1$. The coordinate surfaces with constant ξ are cylinders of elliptic cross-section with foci at A and B . Coordinate surfaces with constant η are hyperbolas

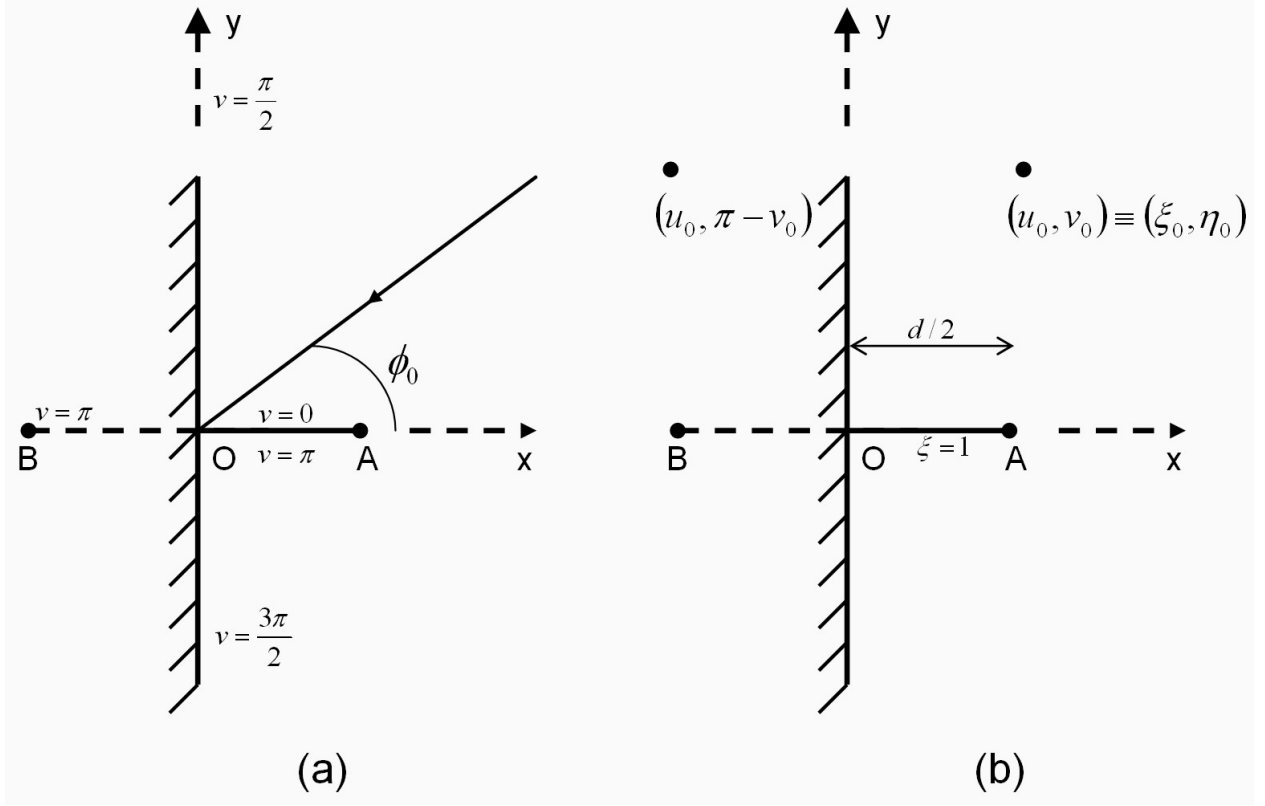


Figure 1: Geometry of the problem: a) plane wave incidence; b) line source incidence.

with the same foci. Observe that, if $\xi = 1$, each value of η defines a point (ξ, η) that falls on the segment of the x axis between the foci A and B. Similarly, if $\eta = 1(-1)$, each value of $\xi > 1$ ($\xi < -1$) defines a point on the segment of the x axis in the interval $x > d/2$ ($x < -d/2$). The positive portion of the y axis corresponds to $v = \pi/2$ and the negative portion, to $v = -\pi/2$. The formulas in subsequent sections contain the dimensionless quantity

$$c = k \frac{d}{2}, \quad (6)$$

where k is the wavenumber and d is the interfocal distance. The height of the blade $OA = d/2$. The inverse transformation from cartesian coordinates (x, y) to elliptic coordinates (ξ, η) is reported here for convenience:

$$\xi = \sqrt{\frac{4(x^2 + y^2) + d^2 + \sqrt{16(x^2 + y^2)^2 + d^4 - 8d^2(x^2 - y^2)}}{2d^2}} \quad (7)$$

and

$$\eta = \frac{2x}{d\xi}. \quad (8)$$

The notation for the Mathieu functions that are used in the following sections is that of [4].

3 Plane wave incidence

3.1 E polarization

The incident field may be written as

$$E_z^i = \exp(jk(x \cos \varphi_0 + y \sin \varphi_0)) = \sqrt{8\pi} \sum_{m=0}^{\infty} j^m \left[\frac{1}{N_m^{(e)}} \text{Re}_m^{(1)}(c, \xi) \text{Se}_m(c, \eta) \text{Se}_m(c, \cos \varphi_0) + \frac{1}{N_m^{(o)}} \text{Ro}_m^{(1)}(c, \xi) \text{So}_m(c, \eta) \text{So}_m(c, \cos \varphi_0) \right] \quad (9)$$

The total geometrical optics (incident plus reflected) field in the absence of the blade is

$$E_z^i + E_z^r = \sqrt{8\pi} \sum_{m=0}^{\infty} j^m \left[\frac{1 - (-1)^m}{N_m^{(e)}} \text{Re}_m^{(1)}(c, \xi) \text{Se}_m(c, \eta) \text{Se}_m(c, \cos \varphi_0) + \frac{1 + (-1)^m}{N_m^{(o)}} \text{Ro}_m^{(1)}(c, \xi) \text{So}_m(c, \eta) \text{So}_m(c, \cos \varphi_0) \right] \quad (10)$$

The scattered field due to the presence of the blade that obeys the two-dimensional radiation condition is written as

$$E_z^s = \sqrt{8\pi} \sum_{m=0}^{\infty} j^m \left[\frac{a_m^{(e)}}{N_m^{(e)}} \text{Re}_m^{(4)}(c, \xi) \text{Se}_m(c, \eta) \text{Se}_m(c, \cos \varphi_0) + \frac{a_m^{(o)}}{N_m^{(o)}} \text{Ro}_m^{(4)}(c, \xi) \text{So}_m(c, \eta) \text{So}_m(c, \cos \varphi_0) \right] \quad (11)$$

The modal coefficients $a_m^{(e)}$ and $a_m^{(o)}$ are obtained by applying the boundary conditions on the plane $x = 0$ and along the blade $\xi = 1$:

$$a_{2l}^{(e)} = a_{2l}^{(o)} = a_{2l+1}^{(o)} = 0, \quad (12)$$

$$a_{2l+1}^{(e)} = -2 \frac{\text{Re}_{2l+1}^{(1)}(c, 1)}{\text{Re}_{2l+1}^{(4)}(c, 1)}. \quad (13)$$

Therefore, the perturbation field E_z^s is given by

$$E_z^s = -j2\sqrt{8\pi} \sum_{l=0}^{\infty} \frac{(-1)^l \text{Re}_{2l+1}^{(1)}(c, 1)}{N_{2l+1}^{(e)} \text{Re}_{2l+1}^{(4)}(c, 1)} \text{Re}_{2l+1}^{(4)}(c, \xi) \text{Se}_{2l+1}(c, \eta) \text{Se}_{2l+1}(c, \cos \varphi_0), \quad (14)$$

and the normalized bistatic cross-section (see [5]) is

$$\frac{\sigma_E(\varphi)}{\lambda} = 32\pi \left| \sum_{l=0}^{\infty} \frac{\text{Re}_{2l+1}^{(1)}(c, 1)}{N_{2l+1}^{(e)} \text{Re}_{2l+1}^{(4)}(c, 1)} \text{Se}_{2l+1}(c, \cos \varphi_0) \text{Se}_{2l+1}(c, \cos \varphi) \right|^2; \quad (15)$$

in particular, for backscattering ($\varphi = \varphi_0$):

$$\frac{\sigma_E^{b.s.}(\varphi)}{\lambda} = 32\pi \left| \sum_{l=0}^{\infty} \frac{\text{Re}_{2l+1}^{(1)}(c, 1)}{N_{2l+1}^{(e)} \text{Re}_{2l+1}^{(4)}(c, 1)} (\text{Se}_{2l+1}(c, \cos \varphi_0))^2 \right|^2 \quad (16)$$

3.2 H polarization

The incident field is

$$H_z^i = \exp(jk(x \cos \varphi_0 + y \sin \varphi_0)) = \sqrt{8\pi} \sum_{m=0}^{\infty} j^m \left[\frac{1}{N_m^{(e)}} \text{Re}_m^{(1)}(c, \xi) \text{Se}_m(c, \eta) \text{Se}_m(c, \cos \varphi_0) + \frac{1}{N_m^{(o)}} \text{Ro}_m^{(1)}(c, \xi) \text{So}_m(c, \eta) \text{So}_m(c, \cos \varphi_0) \right]. \quad (17)$$

The total geometrical optics field (incident plus reflected) in the absence of the blade is

$$H_z^i + H_z^r = \sqrt{8\pi} \sum_{m=0}^{\infty} j^m \left[\frac{1 + (-1)^m}{N_m^{(e)}} \text{Re}_m^{(1)}(c, \xi) \text{Se}_m(c, \eta) \text{Se}_m(c, \cos \varphi_0) + \frac{1 - (-1)^m}{N_m^{(o)}} \text{Ro}_m^{(1)}(c, \xi) \text{So}_m(c, \eta) \text{So}_m(c, \cos \varphi_0) \right]. \quad (18)$$

The perturbation field due to the presence of the blade, that obeys the two-dimensional radiation condition is written as

$$H_z^s = \sqrt{8\pi} \sum_{m=0}^{\infty} j^m \left[\frac{b_m^{(e)}}{N_m^{(e)}} \text{Re}_m^{(4)}(c, \xi) \text{Se}_m(c, \eta) \text{Se}_m(c, \cos \varphi_0) + \frac{b_m^{(o)}}{N_m^{(o)}} \text{Ro}_m^{(4)}(c, \xi) \text{So}_m(c, \eta) \text{So}_m(c, \cos \varphi_0) \right]. \quad (19)$$

The modal coefficients $b_m^{(e)}$ and $b_m^{(o)}$ are obtained by applying the boundary conditions on the plane $x = 0$ and along the blade $\xi = 1$:

$$b_{2l}^{(e)} = b_{2l+1}^{(e)} = b_{2l}^{(o)} = 0, \quad (20)$$

$$b_{2l+1}^{(o)} = -2 \frac{\text{Ro}_{2l+1}^{(1)'}(c, 1)}{\text{Ro}_{2l+1}^{(4)'}(c, 1)}. \quad (21)$$

Therefore, the perturbation field H_z^s is given by

$$H_z^s = -j2\sqrt{8\pi} \sum_{l=0}^{\infty} \frac{(-1)^l \text{Ro}_{2l+1}^{(1)'}(c, 1)}{N_{2l+1}^{(o)} \text{Ro}_{2l+1}^{(4)'}(c, 1)} \text{Ro}_{2l+1}^{(4)}(c, \xi) \text{So}_{2l+1}(c, \eta) \text{So}_{2l+1}(c, \cos \varphi_0), \quad (22)$$

and the normalized bistatic cross-section is given by

$$\frac{\sigma_H(\varphi)}{\lambda} = 32\pi \left| \sum_{l=0}^{\infty} \frac{\text{Ro}_{2l+1}^{(1)'}(c, 1)}{N_{2l+1}^{(o)} \text{Ro}_{2l+1}^{(4)'}(c, 1)} \text{So}_{2l+1}(c, \cos \varphi_0) \text{So}_{2l+1}(c, \cos \varphi) \right|^2; \quad (23)$$

in particular, for backscattering ($\varphi = \varphi_0$):

$$\frac{\sigma_H^{b.s.}(\varphi)}{\lambda} = 32\pi \left| \sum_{l=0}^{\infty} \frac{\text{Ro}_{2l+1}^{(1)'}(c, 1)}{N_{2l+1}^{(o)} \text{Ro}_{2l+1}^{(4)'}(c, 1)} (\text{So}_{2l+1}(c, \cos \varphi_0))^2 \right|^2. \quad (24)$$

4 Line source incidence

4.1 E polarization

Consider a line source parallel to the z -axis and located at the elliptic coordinates $(\xi_0, \eta_0) = (u_0, v_0) = (x_0, y_0)$, whose primary electric field is

$$\underline{E}^i = \hat{z} E_z^i = \hat{z} H_0^{(2)}(kR), \quad (25)$$

where

$$R = \sqrt{(x - x_0)^2 + (y - y_0)^2} \quad (26)$$

is the distance between the line source and the observation point $(x, y) \equiv (u, v) \equiv (\xi, \eta)$. The incident field may be expanded in a series of elliptic cylinder functions [1]:

$$E_z^i = H_0^{(2)}(kr) = 4 \sum_{m=0}^{\infty} \left[\frac{1}{N_m^{(e)}} \text{Re}_m^{(1)}(c, \xi_{<}) \text{Re}_m^{(4)}(c, \xi_{>}) \text{Se}_m(c, \eta_0) \text{Se}_m(c, \eta) + \frac{1}{N_m^{(o)}} \text{Ro}_m^{(1)}(c, \xi_{<}) \text{Ro}_m^{(4)}(c, \xi_{>}) \text{So}_m(c, \eta_0) \text{So}_m(c, \eta) \right], \quad (27)$$

where $\xi_{<}(\xi_{>})$ means the smaller (larger) between ξ_o and ξ . Initially, one assumes that the ridge is absent so that the reflected field is given by an expression similar to (27), where the image of the source is located at $(-x_0, y_0) \equiv (u_0, \pi - v_0)$, as shown in Fig. 1. The total geometrical optics (incident plus reflected) field is

$$E_z^i + E_z^r = 8 \sum_{l=0}^{\infty} \left[\frac{\text{Re}_{2l+1}^{(1)}(c, \xi_{<}) \text{Re}_{2l+1}^{(4)}(c, \xi_{>})}{N_{2l+1}^{(e)}} \text{Se}_{2l+1}(c, \eta_0) \text{Se}_{2l+1}(c, \eta) + \frac{\text{Ro}_{2l}^{(1)}(c, \xi_{<}) \text{Ro}_{2l}^{(4)}(c, \xi_{>})}{N_{2l}^{(o)}} \text{So}_{2l}(c, \eta_0) \text{So}_{2l}(c, \eta) \right]. \quad (28)$$

With the ridge present, the total field is given by

$$E_z = E_z^i + E_z^r + E_z^s \quad (29)$$

where the scattered field E_z^s represents a perturbation to the geometrical optics field (28) and is written as:

$$E_z^s = 8 \sum_{l=0}^{\infty} \left[\frac{A_{2l+1}^{(e)}}{N_{2l+1}^{(e)}} \text{Re}_{2l+1}^{(4)}(c, \xi) \text{Se}_{2l+1}(c, \eta_0) \text{Se}_{2l+1}(c, \eta) + \frac{A_{2l}^{(o)}}{N_m^{(o)}} \text{Ro}_{2l}^{(4)}(c, \xi) \text{So}_{2l}(c, \eta_0) \text{So}_{2l}(c, \eta) \right] \quad (30)$$

The modal coefficients $A_{2l+1}^{(e)}$ and $A_{2l}^{(o)}$ are obtained by applying the boundary conditions along the metal surfaces using the properties reported in [5], [6], which yield

$$A_{2l+1}^{(e)} = -\frac{\text{Re}_{2l+1}^{(1)}(c, 1) \text{Re}_{2l+1}^{(4)}(c, \xi_0)}{\text{Re}_{2l+1}^{(4)}(c, 1)}, \quad (31)$$

$$A_{2l}^{(o)} = 0. \quad (32)$$

4.2 H polarization

The incident field is given by

$$H_z^i = H_0^{(2)}(kr) = 4 \sum_{m=0}^{\infty} \left[\frac{1}{N_m^{(e)}} \text{Re}_m^{(1)}(c, \xi_{<}) \text{Re}_m^{(4)}(c, \xi_{>}) \text{Se}_m(c, \eta_0) \text{Se}_m(c, \eta) + \frac{1}{N_m^{(o)}} \text{Ro}_m^{(1)}(c, \xi_{<}) \text{Ro}_m^{(4)}(c, \xi_{>}) \text{So}_m(c, \eta_0) \text{So}_m(c, \eta) \right]. \quad (33)$$

Assuming that the ridge is absent, the application of the method of images yields the following expression for the total geometrical optics field

$$H_z^i + H_z^r = 8 \sum_{l=0}^{\infty} \left[\frac{\text{Re}_{2l}^{(1)}(c, \xi_{<}) \text{Re}_{2l}^{(4)}(c, \xi_{>})}{N_{2l}^{(e)}} \text{Se}_{2l}(c, \eta_0) \text{Se}_{2l}(c, \eta) + \frac{\text{Ro}_{2l+1}^{(1)}(c, \xi_{<}) \text{Ro}_{2l+1}^{(4)}(c, \xi_{>})}{N_{2l+1}^{(o)}} \text{So}_{2l+1}(c, \eta_0) \text{So}_{2l+1}(c, \eta) \right]. \quad (34)$$

With the ridge present, the total field is given by

$$H_z = H_z^i + H_z^r + H_z^s \quad (35)$$

where the scattered field H_z^s represents a perturbation to the geometrical optics field (34) and is written as:

$$H_z^s = 8 \sum_{l=0}^{\infty} \left[\frac{B_{2l}^{(e)}}{N_{2l}^{(e)}} \text{Re}_{2l}^{(4)}(c, \xi) \text{Se}_{2l}(c, \eta_0) \text{Se}_{2l}(c, \eta) + \frac{B_{2l+1}^{(o)}}{N_{2l+1}^{(o)}} \text{Ro}_{2l+1}^{(4)}(c, \xi) \text{So}_{2l+1}(c, \eta_0) \text{So}_{2l+1}(c, \eta) \right]. \quad (36)$$

The application of the boundary conditions leads to

$$B_{2l}^{(e)} = 0, \quad (37)$$

$$B_{2l+1}^{(o)} = -\frac{\text{Ro}_{2l+1}^{(1)'}(c, 1) \text{Ro}_{2l+1}^{(4)}(c, \xi_0)}{\text{Ro}_{2l+1}^{(4)'}(c, 1)}. \quad (38)$$

5 Comparison with high-frequency approximations

The exact results derived in the previous sections are evaluated using subroutines described in [7] and the acceleration technique presented in [8].

A high-frequency approximation for the field scattered by the blade in the presence of the ground plane is obtained assuming that the ground plane is removed, thus leaving the strip AB shown in Fig. 1. The total field is computed considering the incident field, its image with respect to the initial location of the ground plane and the single diffraction contributions from the edges of the strip. The diffraction by each edge is evaluated by taking the exact scattering by a PEC half-plane and then inserting the asymptotic approximations for the Fresnel integrals. As such, it is a first-order high-frequency result that does not take into account the interaction between the edges. Hence, the approximated expression for the backscattered electric field is:

$$E_z^{b.s.} \approx \frac{\exp(-jk\rho + j\pi/4)}{\sqrt{2\pi k\rho}} \{ \exp(-j2c \cos \varphi_0) [2c \sin \varphi_0 + C_E(c, \varphi_0)] \}, \quad (39)$$

where

$$C_E(c, \varphi_0) = j \cos(2c \cos \varphi_0) - \left[\frac{\sin(2c \cos \varphi_0)}{\cos \varphi_0} + \frac{j}{\sin \varphi_0} \right]. \quad (40)$$

The approximated monostatic radar cross section is

$$\frac{\sigma_E^{b.s.}}{\lambda} = \frac{1}{2\pi} |2c \sin \varphi_0 + C_E(c, \varphi_0)|^2. \quad (41)$$

Fig. 2 shows the comparison between the exact result given in (16) and the high-frequency result given in (41). Only values of φ_0 that satisfy $10^0 < \varphi_0 < 80^0$ are considered because the approximation (41) requires

$$\sqrt{2k\rho} \left\{ \begin{array}{c} \sin \\ \cos \end{array} \right\} \varphi_0 \gg 1 \quad (42)$$

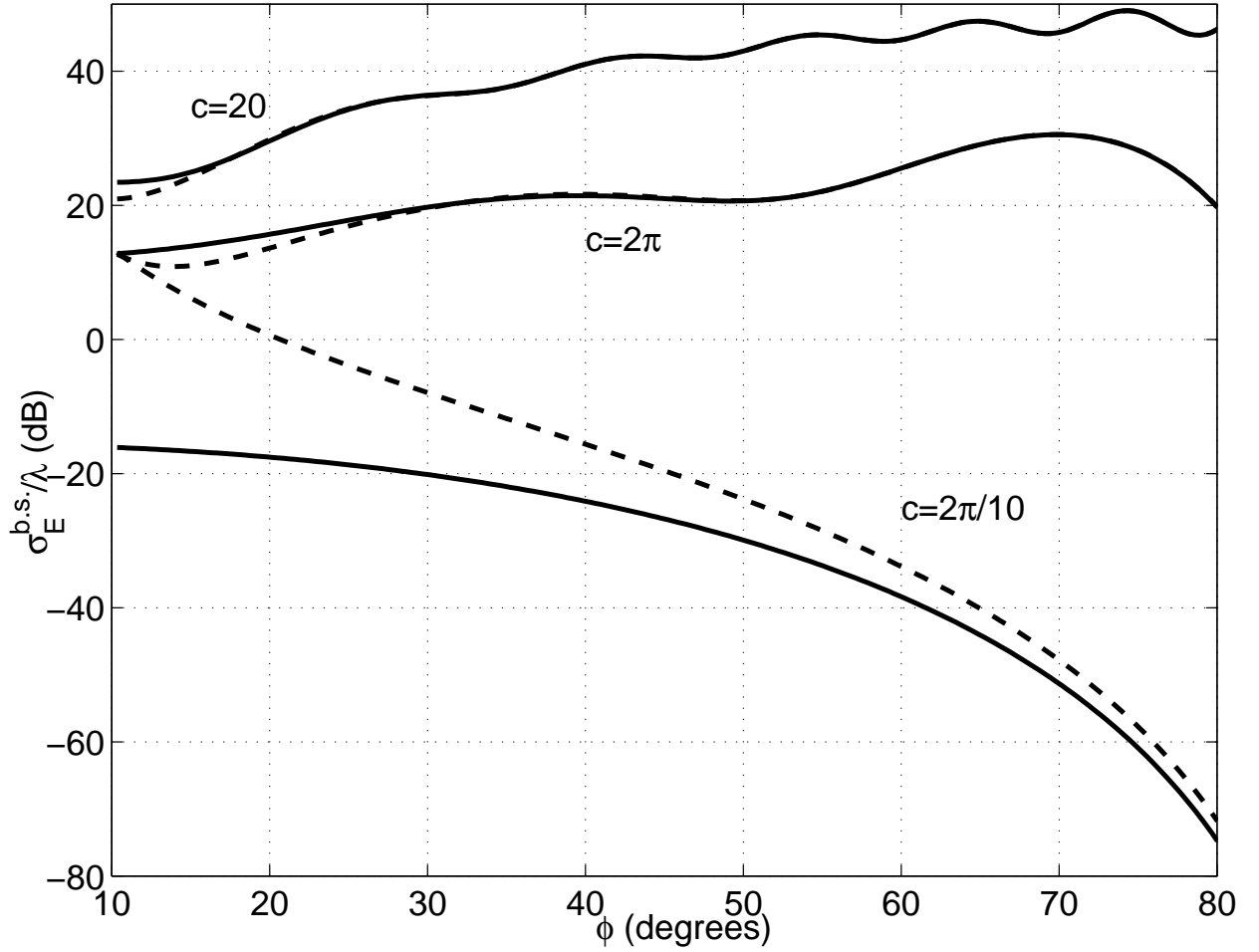


Figure 2: Comparison of the monostatic RCS, in dB, computed using the exact analytical formula (16) and the high-frequency approximation (41) for an E-polarized incident plane wave. The solid lines correspond to the exact solution while the dashed lines represent the high-frequency approximation.

$\sqrt{2k\rho} \sin \varphi_0 \gg 1$. One observes that for small values of c the difference between the two curves is as large as approximately 15 dB, while the agreement significantly improves for larger values of c .

It is also possible to obtain an approximated expression in the case of H polarization. The result for the backscattered magnetic field is:

$$H_z^{b.s.} \approx \frac{\exp(-jk\rho + j\pi/4)}{\sqrt{2\pi k\rho}} \{ \exp(-j2c \cos \varphi_0) [2c \sin \varphi_0 + C_H(c, \varphi_0)] \}, \quad (43)$$

where

$$C_H(c, \varphi_0) = j \cos(2c \cos \varphi_0) + \left[\frac{\sin(2c \cos \varphi_0)}{\cos \varphi_0} + \frac{j}{\sin \varphi_0} \right]. \quad (44)$$

The approximated monostatic radar cross section is

$$\frac{\sigma_H^{b.s.}}{\lambda} = \frac{1}{2\pi} |2c \sin \varphi_0 + C_H(c, \varphi_0)|^2. \quad (45)$$

Fig. 3 shows the comparison between the exact result (24) and the approximate result (45). In

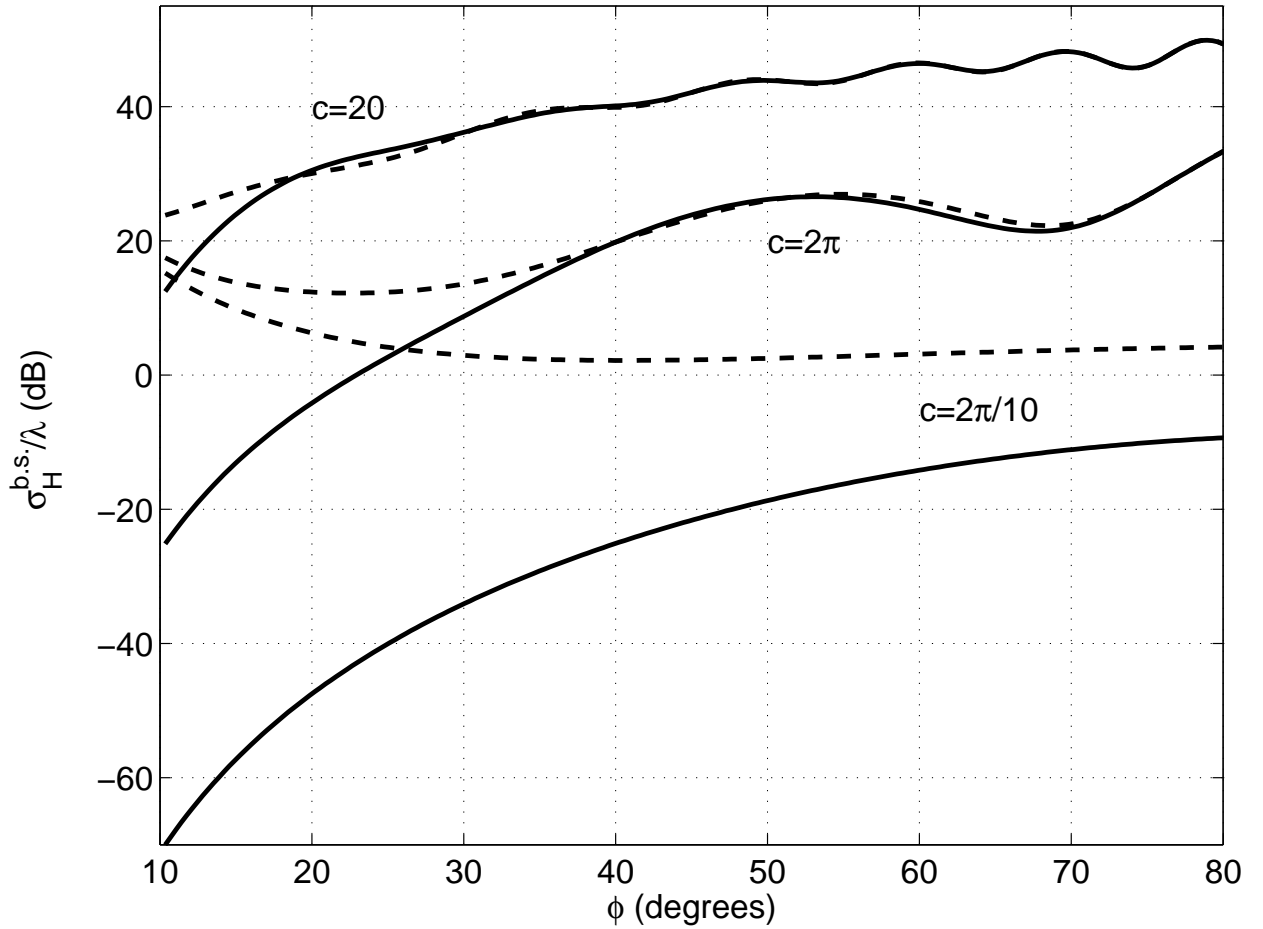


Figure 3: Comparison of the monostatic RCS, in dB, computed using the exact analytical formula (24) and the high-frequency approximation (45) for an H-polarized incident plane wave. The solid lines correspond to the exact solution while the dashed lines represent the high-frequency approximation.

the case of H polarization, the discrepancy between the approximated (45) and exact (24) results

is larger then for the corresponding values of c in the case of E polarization. This may be due to a traveling wave between the edges that is present for H polarization but not for E polarization. However, as expected, the agreement improves for larger values of c .

6 Comparison with the uniform theory of diffraction

The exact total fields given by (29) and (35) for the E and H polarizations, respectively, are approximated using the uniform theory of diffraction (UTD) [9], [10]. Similar to the high-frequency approximation of section 5, the UTD approximation is obtained by removing the metal plane $x = 0$, introducing the image OB of the segment OA and the image of the source. As a result, the contribution of the source towards the observation point will consist of two diffracted rays and, if a line of sight exists, a direct ray, as shown in Fig. 4. A similar set of rays must be considered also for the image of the source.

Figure 5 shows the results for the comparison when the incidence is an E-polarized plane wave. The curves for the exact and UTD results overlap for the values considered for c along the observation segment that is located close to the blade at a distance of only one wavelength; in fact, a small difference is noticeable only when c is quite small, as in Fig. 5a.

Fig. 6 shows the results for line source incidence and E polarization. In all three cases, the agreement between the exact theory and the high-frequency method is very good. In fact, one can hardly recognize that for each value of c two curves are actually overlapped, even though the distances among source, ridge, and observation point are only a few wavelengths.

Fig. 7 shows the results for line source incidence and H polarization. Unlike the case of E polarization, it is possible to observe the difference between the solid and dashed line. This difference is particularly noticeable for the case $c = 1$, which correspond to a ridge that is shorter than one wavelength. When the value of c increases, the UTD approximation provides results that are closer to the exact ones.

7 Comparison with measurements

Experiments were also conducted to compare the theoretical results given by (29) and (35) with measurements. The measurements were taken in the anechoic room at the University of Illinois at Chicago and the antennas used in the experiments were the same used for the experiments described in [11] and [12]. Fig. 8 shows the results for the E-polarization case. One can observe a very good agreement when the distance of the observation point from the ground plane exceeds approximately 12λ . When the observation point is closer to the the ground plane the agreement is not so good and this behavior is attributed to an interaction between the antenna used in the experiment and the metallic plane. Similar observations apply to Fig. 9.

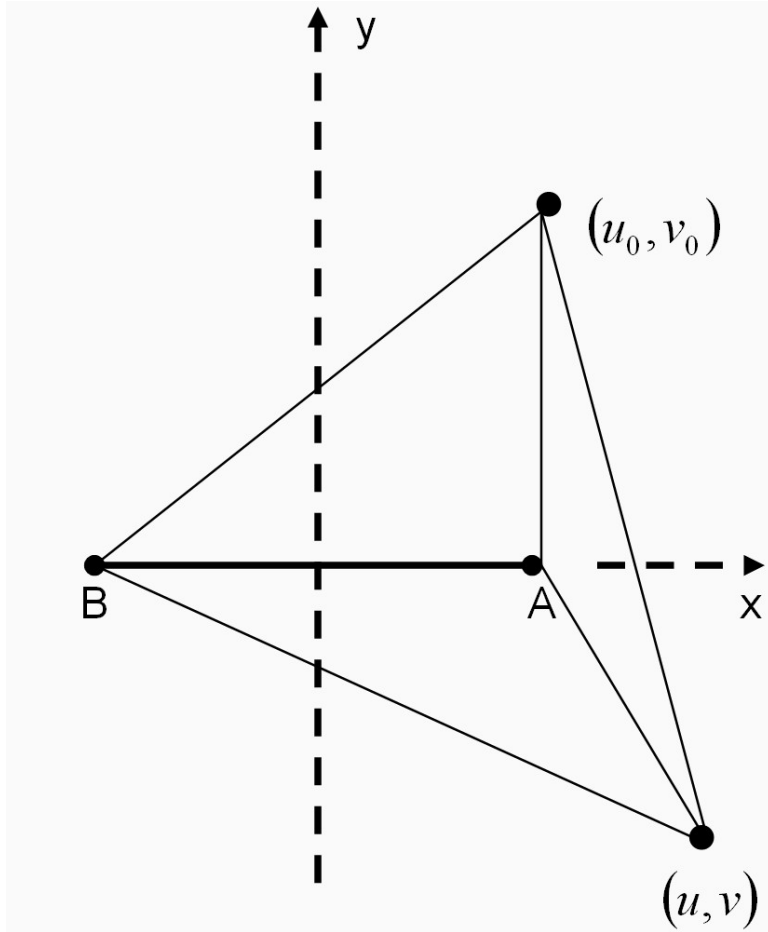


Figure 4: Geometry of the problem without the PEC plane. This figure only shows the contribution from the original source, but another set of three rays needs to be considered to account for the total field.

8 Conclusion

The results obtained in this work are important because they present exact solutions to a new boundary value problem, thus enriching the list of geometries for known canonical solutions. Other exact solutions to new boundary value problems have been presented in [5] and [6]. Additionally, the comparisons with high-frequency approximations and measurements confirm the validity of the theoretical predictions. We could have introduced higher-order terms in the high-frequency approximations; however, we wanted to show that the first-order terms can provide accurate results, even with relatively small values of c .

It should be noted that the results obtained in this work can also be derived from the results for a PEC strip, in free space, by utilizing the method of images.

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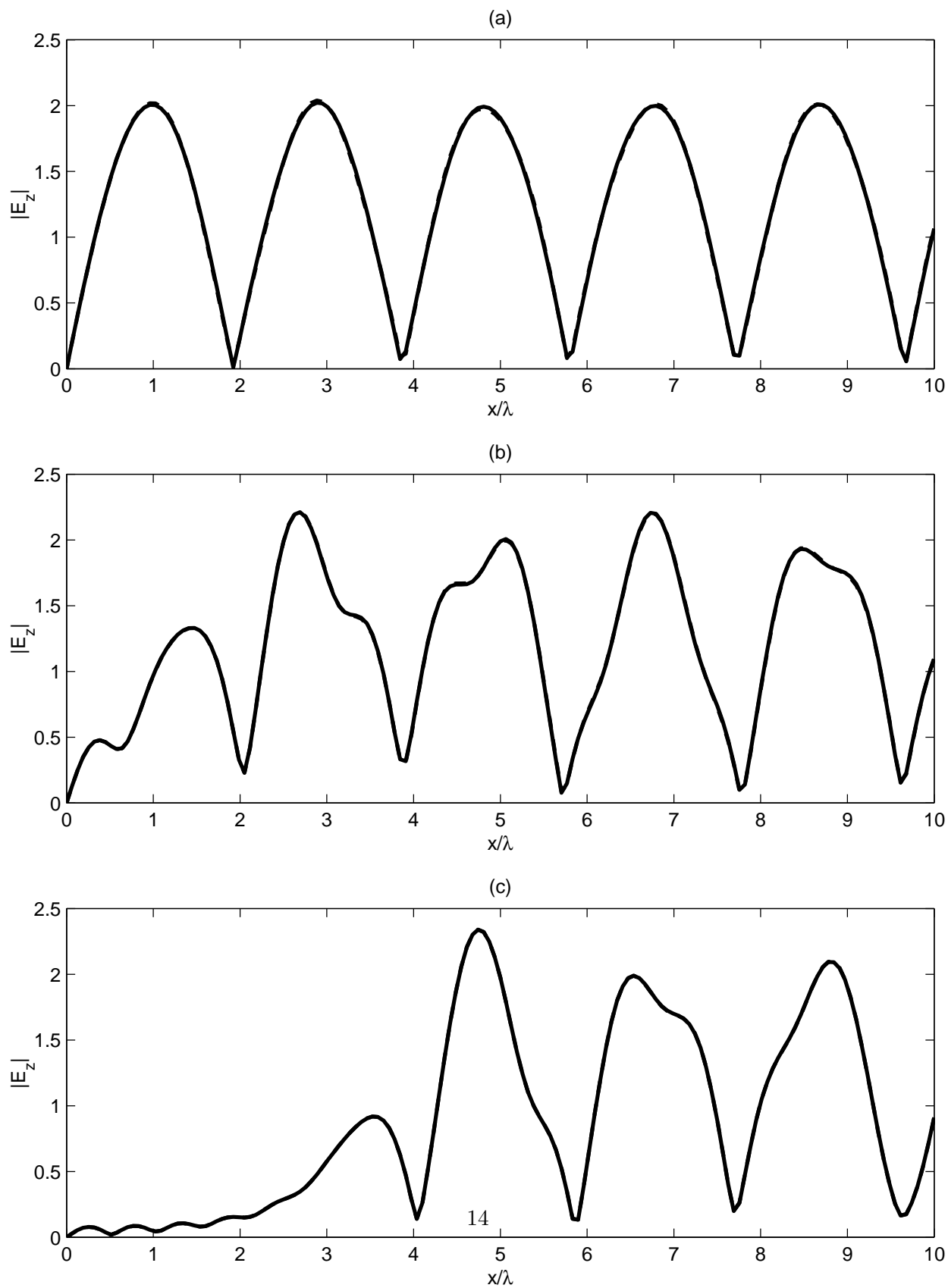


Figure 5: E-polarized plane wave incident at an angle $\varphi_0 = 75^\circ$. The total normalized field $|E_z|$ is computed along the line $(0 \leq x \leq 10\lambda, y = -\lambda)$. The results correspond to: (a) $c = 2\pi/10$; (b) $c = 2\pi$; and $c = 20$.

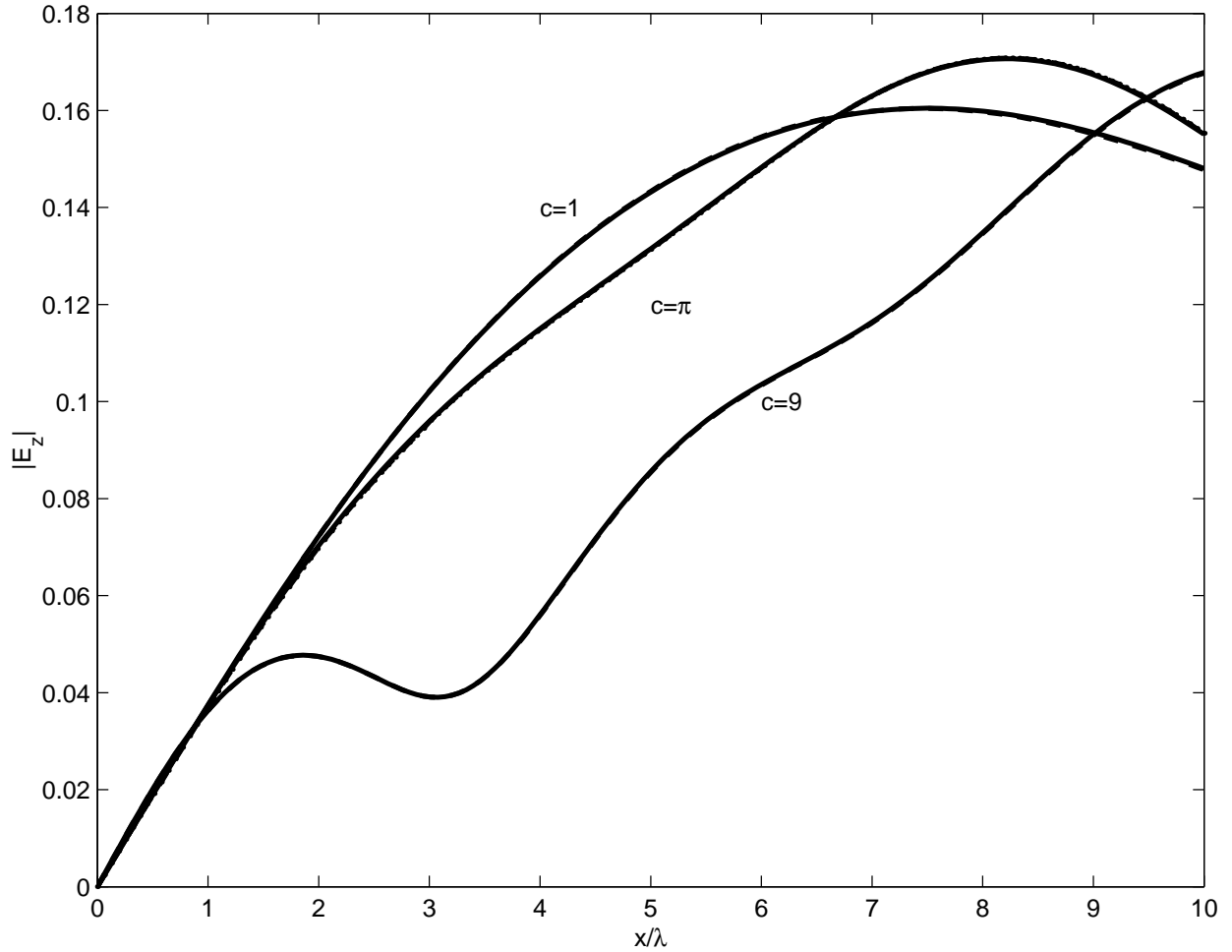


Figure 6: E polarization. Total normalized field $|E_z|$ due to a line source located at $(x_0 = \lambda/2, y_0 = 4\lambda)$ when the observation point moves along the line $(0 \leq x \leq 10\lambda, y = -10\lambda)$. The solid lines represent the exact results, while the dashed lines represent the UTD approximation. Because of the close agreement between the exact results and the UTD approximation, the two sets of curves overlap very well and any difference is hardly noticeable. The computed results correspond to: 1) $c = 1$; 2) $c = \pi$; and 3) $c = 9$.

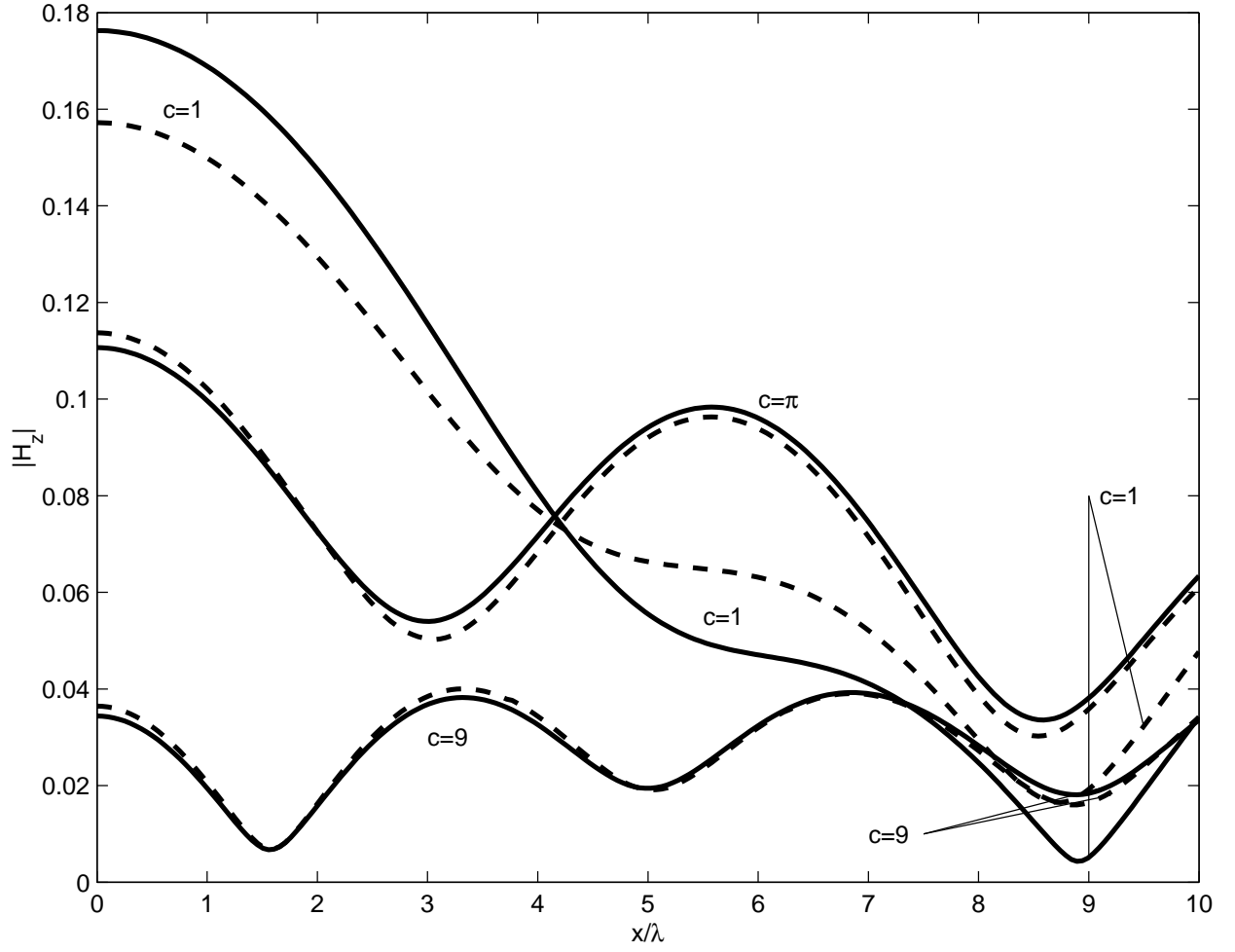


Figure 7: H polarization. Total normalized field H_z due to a line source located at $(x_0 = \lambda/2, y_0 = 4\lambda)$ when the observation point moves along the line $(0 \leq x \leq 10\lambda, y = -10\lambda)$. The solid lines represent the exact results, while the dashed lines represent the UTD approximation. The computed results correspond to: 1) $c = 1$; 2) $c = \pi$; and 3) $c = 9$.

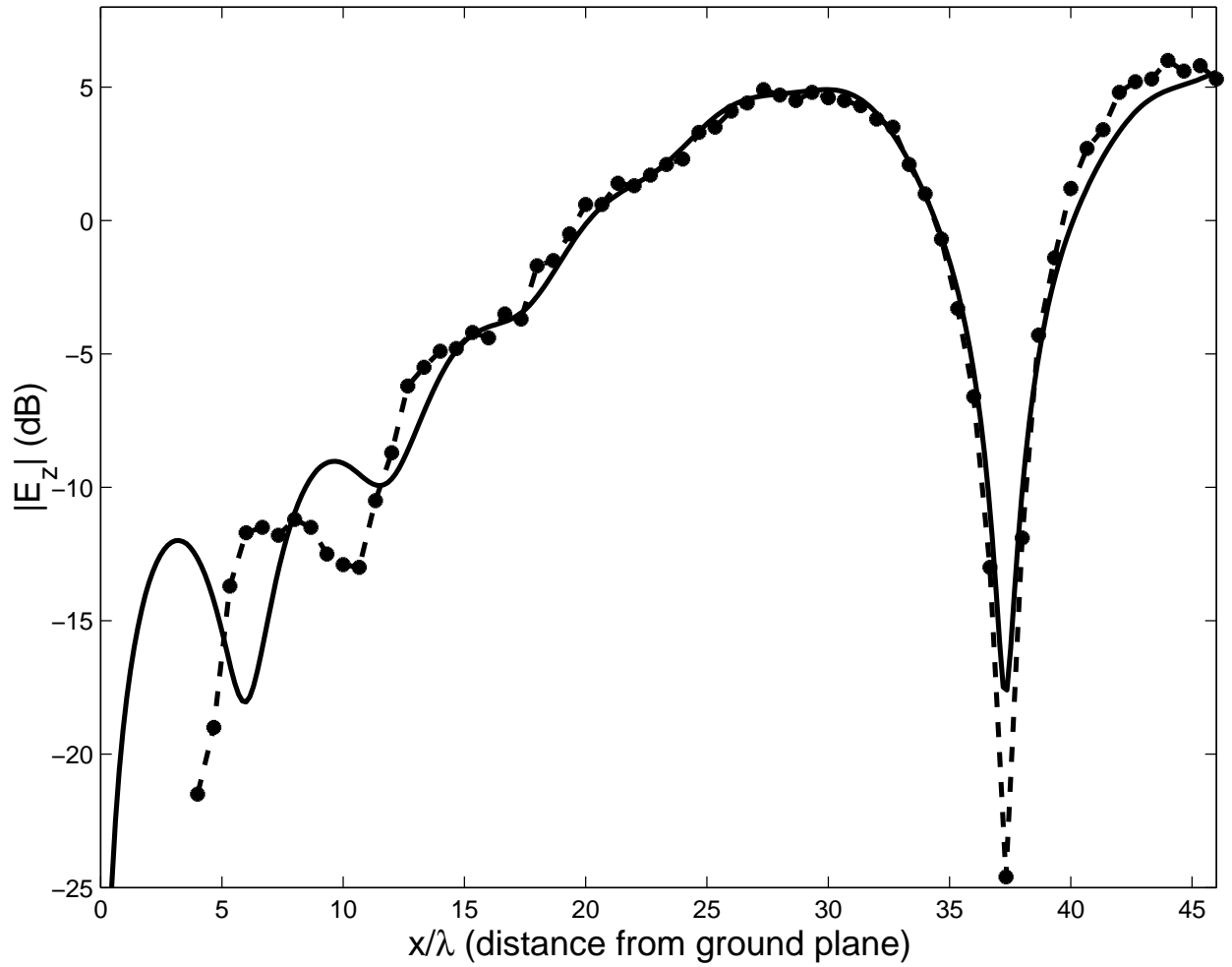


Figure 8: E polarization. Normalized value of the total field $|E_z|$ due to a line source located at $(x_0 = 6.25\lambda, y_0 = 115\lambda)$ when the observation point moves along the line $(0 \leq x \leq 46\lambda, y = -114.17\lambda)$. The solid line represents the exact results, while the dashed line represents the measurements. These results correspond to $c = 52$.

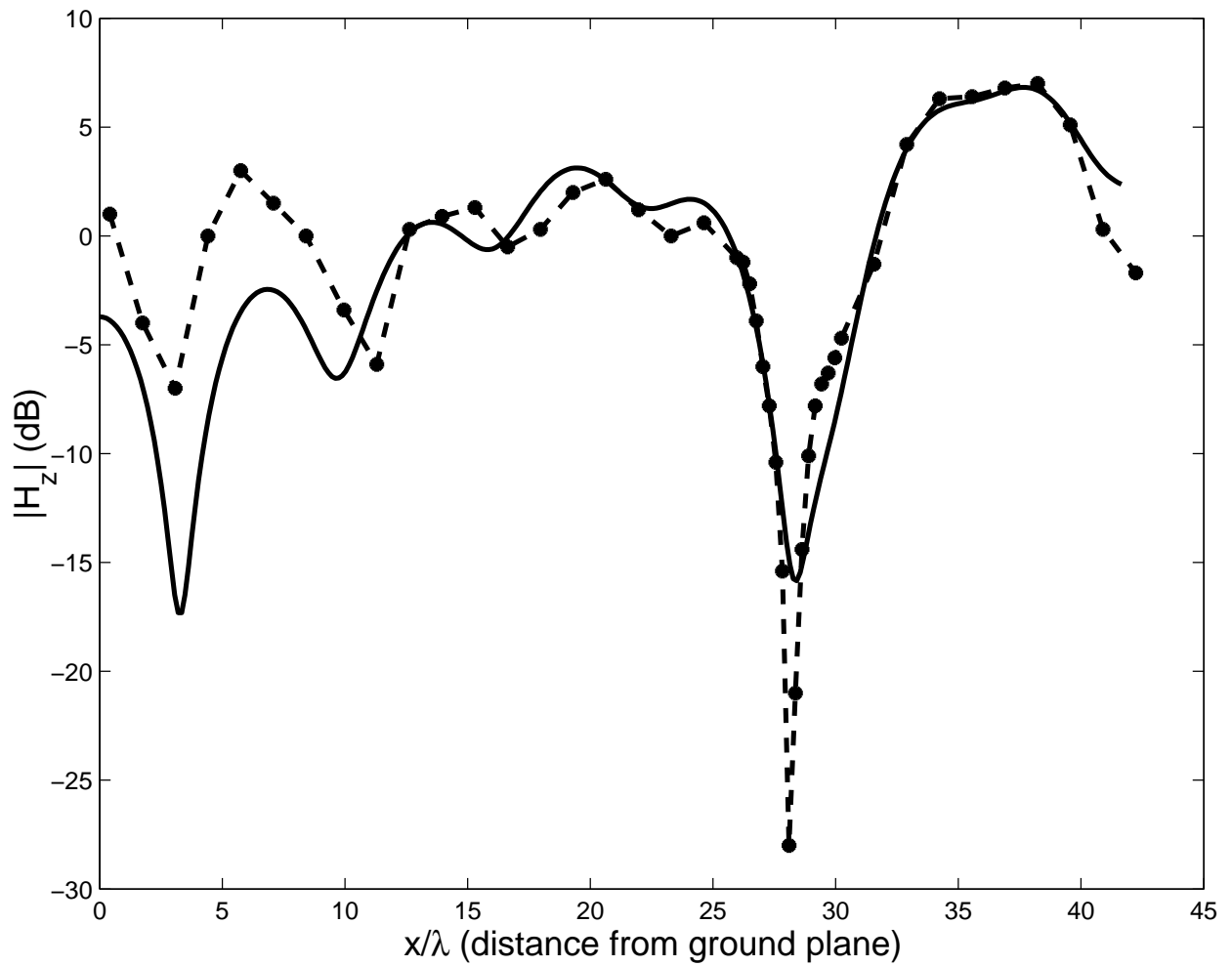


Figure 9: H polarization. Normalized value of the total field $|H_z|$ due to a line source located at $(x_0 = 6.25\lambda, y_0 = 115\lambda)$ when the observation point moves along the line $(0 \leq x \leq 42\lambda, y = -114.17\lambda)$. The solid line represents the exact results, while the dashed lines represent the measurements. These results correspond to $c = 52$.