Positioning and Latinas/os

A Study of Small-Group Interactions in Mathematics

BY

CARLOS ALFONSO LÓPEZ LEIVA
P.E.M., Universidad Rafael Landívar, Quetzaltenango, Guatemala, 1993
Lic., Universidad Rafael Landívar, Guatemala, 2001
M.Ed., University of Illinois at Chicago, Chicago, 2006

THESIS

Submitted as a partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Curriculum and Instruction (Curriculum Studies)
in the Graduate College of the
University of Illinois at Chicago, 2011

Chicago, Illinois

Defense Committee:

Lena Licón Khisty, Chair and Advisor
Danny B. Martin
Aria Razfar
David Mayrowetz, Educational Policy Studies
Luis C. Moll, University of Arizona
This thesis is dedicated to my parents, Luz del Carmen and Alfonso, who always positioned me as a person, a son, and a friend.
ACKNOWLEDGEMENTS

I would like to thank my advisor, Lena Licón Khisty, who continuously provided support, advice, and challenges to help me move forward.

Thanks also to my dissertation committee members—Aria Razfar, Danny Martin, David Mayrowetz, and Luis Moll—for their timely advice.

For their support as fellow students and researchers as well as friends (cuates), I would like to thank my research fellows Zayoni Torres, Higinio Domínguez, Griselda Velázquez, Craig Willey, Alexander Radosavljević, Angela Thompson, George Gutiérrez, Eugenia Vomvoridi-Ivanović, Kathleen Pitvorec, Hector Morales, Jesús Acosta, and Gabriel Viego.

For inspiring and supporting me to face life, I offer my gratitude to my friends Mónica García, Mario Morán, Magdalena Salazar, Marisel Estrada, Winny Hung, Mauricio Alvarado, to my wonderful sister Luz López de Matul and my nieces Luz María and Liza Rebecca, and especially to Darren Ilett, who supported and struggled with me through this work.

Finally and most especially, I would like to acknowledge both the pre-service teachers (UGs) who were part of Los Rayos de CEMELA because their work made it possible as well as the students who participated in the program and illuminated my day every time I saw them and they shared with me their energy, work, laughter, and cariño.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>A. Background</td>
<td>1</td>
</tr>
<tr>
<td>B. My Journey into the Topic</td>
<td>2</td>
</tr>
<tr>
<td>C. Definition of Terms</td>
<td>4</td>
</tr>
<tr>
<td>D. Focus on Latina/o Bilingual Students</td>
<td>5</td>
</tr>
<tr>
<td>E. Relevance of Social Interaction and Positioning in Mathematics</td>
<td>8</td>
</tr>
<tr>
<td>1. The Social Turn in Mathematics Education and Research</td>
<td>8</td>
</tr>
<tr>
<td>2. Limitations and Breakdowns During Social Interactions</td>
<td>10</td>
</tr>
<tr>
<td>3. Current Related Concerns in Mathematics Education</td>
<td>14</td>
</tr>
<tr>
<td>4. Relevance of Research Context</td>
<td>15</td>
</tr>
<tr>
<td>F. Preliminary Work and Research Questions</td>
<td>15</td>
</tr>
<tr>
<td>G. Summary</td>
<td>17</td>
</tr>
<tr>
<td>II. REVIEW OF RELATED LITERATURE</td>
<td>18</td>
</tr>
<tr>
<td>A. Overview</td>
<td>18</td>
</tr>
<tr>
<td>B. Positioning</td>
<td>18</td>
</tr>
<tr>
<td>C. Participation</td>
<td>23</td>
</tr>
<tr>
<td>D. Studies on the Social Dimension of Learning</td>
<td>25</td>
</tr>
<tr>
<td>E. Review of Related Empirical Research Literature</td>
<td>27</td>
</tr>
<tr>
<td>1. Positioning in Mathematics and Science</td>
<td>27</td>
</tr>
<tr>
<td>2. Social Interactions in Mathematics</td>
<td>31</td>
</tr>
<tr>
<td>3. Social Interactions and Exclusion</td>
<td>34</td>
</tr>
<tr>
<td>F. Summary</td>
<td>34</td>
</tr>
<tr>
<td>III. THEORETICAL FRAME</td>
<td>35</td>
</tr>
<tr>
<td>IV. METHODOLOGY</td>
<td>41</td>
</tr>
<tr>
<td>A. Overview</td>
<td>41</td>
</tr>
<tr>
<td>B. Research Questions</td>
<td>41</td>
</tr>
<tr>
<td>C. Why Qualitative Methods and Case Study?</td>
<td>41</td>
</tr>
<tr>
<td>D. Data Collection and Analytical Processes</td>
<td>44</td>
</tr>
<tr>
<td>1. Participants</td>
<td>44</td>
</tr>
<tr>
<td>2. Site of Data Collection</td>
<td>48</td>
</tr>
<tr>
<td>3. Data Sources</td>
<td>50</td>
</tr>
<tr>
<td>4. Data Organization and Reduction</td>
<td>51</td>
</tr>
<tr>
<td>5. Data Analysis</td>
<td>52</td>
</tr>
<tr>
<td>E. Summary</td>
<td>57</td>
</tr>
<tr>
<td>V. FINDINGS</td>
<td>59</td>
</tr>
<tr>
<td>A. Overview</td>
<td>59</td>
</tr>
<tr>
<td>B. How Does Positioning Develop in Interactions around Problem Solving?</td>
<td>61</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. NAEP 2007, 4&lt;sup&gt;th&lt;/sup&gt;-GRADE SD &amp; ELL STUDENT MATHEMATICAL ACHIEVEMENT LEVEL</td>
<td>7</td>
</tr>
<tr>
<td>II. PROFILES OF STUDENT CASES</td>
<td>47</td>
</tr>
<tr>
<td>III. INSTANCES OF LANGUAGE FUNCTIONS MARKING DIFFERENCES</td>
<td>105</td>
</tr>
<tr>
<td>IV. LANGUAGE PATTERNS AND FUNCTIONS MARKING DIFFERENCES</td>
<td>106</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Lines of power structuring and affecting interactions</td>
<td>37</td>
</tr>
<tr>
<td>2. Immediate and over time processes during social interactions around problem solving in mathematics.</td>
<td>38</td>
</tr>
<tr>
<td>3. Multiple-case design of the study.</td>
<td>42</td>
</tr>
<tr>
<td>4. <em>Los Rayos’</em> timeline and curriculum units</td>
<td>50</td>
</tr>
<tr>
<td>5. What is ½?</td>
<td>82</td>
</tr>
<tr>
<td>6. What is 1/8?</td>
<td>82</td>
</tr>
<tr>
<td>7. What is 3/8?</td>
<td>82</td>
</tr>
<tr>
<td>8. Tiles in El Maga’s hat.</td>
<td>97</td>
</tr>
<tr>
<td>9. Betty’s leadership connected to language use</td>
<td>109</td>
</tr>
<tr>
<td>10. Percentages of students’ positions over time</td>
<td>119</td>
</tr>
<tr>
<td>11. Facilitators’ descriptions of student cases during 1st semester of <em>Los Rayos</em></td>
<td>122</td>
</tr>
<tr>
<td>12. General positions over time by type</td>
<td>124</td>
</tr>
<tr>
<td>13. Factors mediating the positioning process in mathematical activity.</td>
<td>129</td>
</tr>
</tbody>
</table>
# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEMELA</td>
<td>Center for the Mathematics Education of Latinas/os</td>
</tr>
<tr>
<td>CHAT</td>
<td>Cultural Historical Activity Theory</td>
</tr>
<tr>
<td>ELLs</td>
<td>English Language Learners</td>
</tr>
<tr>
<td>MPS</td>
<td>Mathematics Problem Solving</td>
</tr>
<tr>
<td>NAEP</td>
<td>National Assessment of Educational Progress</td>
</tr>
<tr>
<td>NCES</td>
<td>National Center for Educational Statistics</td>
</tr>
<tr>
<td>NCTM</td>
<td>National Council of Teachers of Mathematics</td>
</tr>
<tr>
<td>PT</td>
<td>Positioning Theory</td>
</tr>
<tr>
<td>SD</td>
<td>Student with Disabilities</td>
</tr>
<tr>
<td>US</td>
<td>United States</td>
</tr>
<tr>
<td>UGs</td>
<td>Undergraduate pre-service teachers working in the program with students</td>
</tr>
<tr>
<td>ZPD</td>
<td>Zone of Proximal Development</td>
</tr>
</tbody>
</table>
SUMMARY

This study explores social interaction and mathematics performance, especially regarding how bilingual Latino/a students’ positioning and participation evolved as they interacted with others in mathematics problem solving. Historically, Latinas/os have faced segregation in different ways that affect the quality of their social interactions and, eventually, also their learning. Although great efforts have been made in promoting equitable student participation and improving student-teacher and student-student interactions, there still remains the question of how both equitable and uneven participation evolve in natural settings. Current research has demonstrated the need to document, through longitudinal ethnographic studies, how students’ interactions and social exclusion affect their learning in mathematics (Baumeister & De Walle, 2005; Zevenbergen, 2001). Thus, greater interest has evolved in children’s mathematical discourses, social worlds, cooperative work (Gonzalez, Andrade, Civil, & Moll, 2001), and positional identities (Esmonde, 2009).

From a sociocultural perspective, this study centered on Latina/o students and their access to resources and opportunities to learn mathematics (Gutiérrez, 2008) by exploring how participation in group problem solving affected the “who you are” (positioning), and how this position affected student participation in the same event (Bloome, Carter, Christian, Otto, & Shuart-Faris, 2005). Positioning, described as a discursive act, refers to the roles and actions that participants take up and/or are given when interacting with others (Harré, 2004). Participation refers to people engaging in a social activity in which the interactions, active observations, and shared thinking mediate the accomplishment of the activity, which in turn also transform the individuals (Rogoff, 1990). In essence, participation describes the engagement of individuals in a task and how they relate to one another and the task. These constructs gave rise to the following question:

How do social interactions in small-group work mediate the positioning and participation of bilingual Latina/o students during mathematics problem-solving tasks in an afterschool program?
This study drew on data gathered as part of a larger study, *Los Rayos* (Khisty, 2004), concerned with the language and cultural resources of Latinas/os in mathematics. Ethnographic data of the same students were gathered over three years in an afterschool setting. This presented the opportunity to observe circumstances and students’ characteristics that in regular classrooms may be hidden or not deemed as obvious (McDermott & Varenne, 1998). The data included participant observations and analysis of 1) videotaped student interactions, 2) discourse used by participants in each group, 3) field notes developed by group facilitators (research fellows and pre-service teachers), and 4) relevant student work. I used a multiple-case design (Yin, 2009) of four students that represented contrasting cases (high or low) given their participation status observed in the afterschool and their school grades. I explored their interactions and language use affecting the evolution of their positioning and participation during problem solving.

Results indicated three factors mediating the process of co-constructing one’s own and others’ positions, namely the quality of *attention, alignment*, and *ability* co-constructed among the participants. The quality of these factors, unevenly distributed and negotiated among the group participants, afforded different positions for participants (i.e., powerful, null, powerless, and equalized), which in turn also determined the quality of mathematical and social affordances that participants accessed in the group. Positioning patterns were flexible as all students experienced all types of positions. However, over time these patterns were recurrent and linked to specific students. Greater levels of social support were given to students with higher mathematical performance. The type of tasks and groups in which students interacted also seemed to be factors in the negotiation of positions. Codeswitching was used by bilingual participants around positioning events to mark differences or agreements, and English was used as a way to gain authority.
These results as well as their implications and possible suggestions for instruction and research are discussed in the final chapter. The limitations of the study are twofold: first, the results are constrained by the circumstances of the afterschool (e.g., the study cannot make claims pertaining to a regular classroom); and second, the nature of ethnographic study, with its small number of subjects, limits the extent to which the findings can be generalized. The relevance of this study resides in the exploration of social and academic practices that support Latinos/as’ positioning and participation more consistently and positively, as well as those that may limit their potential and learning opportunities in mathematics.
I. INTRODUCTION

A. Background

“Social relations or relations among people genetically underlie all higher functions and their relationships. Hence one of the basic principles of volition is that of the division of functions among people, the new division into two parts of what is now combined into one. It is the development of a higher mental process in the drama that takes place among people.” (Vygotsky, 1981, p. 163)

Learning is a human endeavor in which people, usually adults and children, come together and interact with one another. Learning has often been considered only from the perspective of cognitive development, which overlooks the interactions that are at its core. Because learning is based on these human interactions, it can be vulnerable to perspectives, actions, statements, and gestures that position one individual differently than another. These positions may provide different levels of power based on assumptions of knowledge or lack of knowledge and thereby affect what and how one learns.

Given the relevance of social interactions in learning and especially in problem solving in mathematics, I have become interested in better understanding the process of positioning, especially as it affects bilingual Latinas/os students in mathematics and particularly when they are engaged in problem solving during small-group interactions. I focused on problem-solving situations due to increased opportunities for interactions among participants, as opposed to times when they are following someone else’s directions, such as in learning mathematical algorithms. Furthermore, the current mathematics reform considers problem solving as the cornerstone of its new direction and student collaboration in small groups as a crucial format of interaction for facilitating learning and meaning making (National Council of Teachers of Mathematics, 2000).

In this chapter, I discuss how I became interested in the phenomena of positioning and participation and why the focus is on Latinas/os. I also present how the focus of this study is
relevant to the field of mathematics education of Latinas/os, not only in its context, but also its relation to current teaching-learning and research interests in the field. I conclude with a delineation of the specific questions this research addresses.

B. My Journey into the Topic

In research and personally, I identify myself as a bilingual (Spanish and English) Latino, *Ladino,* or *Mestizo* (mixture of European and Native American people) Guatemalan man. I am deeply concerned with issues of bilingual teaching and the learning of mathematics among Latinas/os, especially those who are low-achieving and those with disabilities. I also acknowledge the fact that being Latino does not guarantee my automatic understanding of Latino diversity. Nonetheless, as a Latino man in the US, I have had experiences that may relate to those of some other Latinas/os in this country.

In Guatemala, although not rich, I grew up as part of the dominant group (i.e., *Ladinos*). I did not have a racialized experience as many indigenous people do, but I partly did by being gay. I believe that these circumstances made me aware of and concerned about not only how people’s social positions affect their quality of life but also how these social constructions evolve. Growing up, I witnessed societal oppression exerted upon indigenous people such as my friends. They were often ostracized based on linguistic, cultural, or physical features. I believe that these experiences started opening my eyes about differences among people and how political they can be.

As a teacher, I noticed how some of my students wanted to stop speaking their native languages because they felt ashamed of them, given how the languages were socially and politically devalued. Often students with more obvious “indigenous” appearance or last names were made fun of and positioned as less important people. Barrera (1979) describes colonialism as a relationship of domination based on racial lines where a dominant group makes out of its own interests and own
benefits the only leading paradigm for the relationship. Thus, in living our everyday lives, we can embrace colonizing actions, approaches, and ideologies that we have been exposed to and appropriated uncritically in turn. These are continuously active, especially as we interact, and affect how we include, exclude, and position others and ourselves.

When I came to the US 8 years ago, I started understanding what a racialized experience is. I became an “alien” (immigrant person), a person with an accent, a Hispanic, and by looking as I do, some people considered me as “different” and thus as a “dangerous” person (perhaps a mugger or a thief), even without attempting to find out about me. Despite all these experiences, when I started the doctoral program, my interest concerned mathematics learning but not the social dimension of learning. From the beginning of my doctoral studies, I served as a research assistant and coordinator in a mathematics afterschool program, which was created and conducted by Dr. Lena Licón Khisty, the Principal Investigator. The program was part of the Center for the Mathematics Education of Latinos (CEMELA), supported by the National Science Foundation. During this time, we documented all student work. (I will elaborate on this program later in chapter 4 when I describe the context of my study). As I started examining the data that we collected through CEMELA, I became concerned with one student’s difficulty to grasp a mathematical concept. My first thought was to assume that the student’s problem was due to low language proficiency in English. However, as I explored the participation of all the students more deeply, I noticed that English dominant peers participated the most, while this student kept silent. This situation aroused my interest in the interactions among students and how they affected participation. In other words, I realized that language proficiency was not what caused the student’s problem in mathematics. Instead, the circle of interactions had to be more carefully examined and taken into account. My own personal experiences plus my observations in the afterschool have led me to realize that, during interactions
in mathematics, the social dimension of learning plays as crucial a role as content and that this warrants further exploration.

I present the first two points and then briefly describe the phenomena of positioning and participation and analyze their growing interest in the field of mathematics education through connections with social psychology and sociocultural approaches.

C. Definition of Terms

In this document, I use several critical terms. In this section, I will explain what I mean by each one. I often use the term bilingual (Spanish-English) Latina/o students. I understand the term bilingual as denoting a continuum of proficiency in speaking and writing two languages. The two languages are at the opposite points of the continuum, with the middle representing a balanced fluency between the two languages (van Lier, 2004). Other terms are commonly used to describe the bilingual population, for example: English Language Learners, Language Minority Students, Limited English Proficient Students, and Second Language Speakers. In the US context, these terms often acknowledge the process of learning English as a second language. At times, students can grow out of these terms as they improve their fluency in English. Other terms simply mark these students as different or having a limitation without explicitly acknowledging what these students really know. For these reasons, I think that these terms only present a narrow description of bilingual students that does not capitalize equally on both of their linguistic sources throughout and after the process of learning a new language. Without dismissing the need to acknowledge language proficiency, I use the term bilingual and note different levels of proficiency in either language. During the rest of my discussion in this document, I include these terms along with other authors’ discussions and findings.
Other critical terms in this study include *positioning* and *participation*. I will describe both terms thoroughly in chapter II and at different points in this document. But as an introduction, I will briefly define them here. The former term refers to a discursive process that describes how individuals relate to one another and the task during the event. The latter term describes the engagement of individuals in a task and with each other regarding the amount and quality of their opportunities for engagement.

D. **Focus on Latina/o Bilingual Students**

All students in this study demonstrated proficiency in both languages (English and Spanish). Therefore, I refer to them as bilingual Latino students. The population of Latinas/os in the United States (US) as of 2010 was about 50.5 million, which comprises 16% of the total population (Ennis, Ríos-Vargas, & Albert, 2011). Latinas/os represent a diverse group: White, Black, Native American, and other races, but they are commonly associated with speaking Spanish, which represents a narrow perspective (Humes, Jones, & Ramírez, 2011), since Latinas/os may speak other languages and they may or may not speak Spanish. In education, Latina/o students comprise over 22% of the total student population in the US as of 2009 (National Center for Educational Statistics, NCES, 2011). Over half of Latina/o students speak English at home (Llagas & Snyder, 2003). Yet Latina/o students have been considered at-risk because they have historically fallen outside of mainstream American schools in aspects such as race, ethnicity, language, culture, family, and socio-economic status (Gonzalez, Brusca-Vega, & Yawkey, 1996). Attention to their social, cultural, intellectual, and primary linguistic resources (Spanish or not) will not only promote cultural cohesiveness and support their understanding and learning processes, but will also uncover these students’ hidden talents, oral histories, skills, and latent abilities (Moll, 1992). Educational reports describe that Hispanic students have higher high school dropout rates than White or Black
students (Llagas & Snyder, 2003). Using national statistics, Flores (2007) shows that Latino and low-income students are less likely to have access to experienced and qualified teachers, more likely to face low expectations, and less likely to receive equitable funding per student. Similarly, most Latinas/os attend schools where minorities are the majority of the student body (Llagas & Snyder, 2003). In fact, Latinos are more likely than any other ethnic group in the US to attend hyper-segregated schools, which may inhibit their development of the norms and expectations of the broader society. Yet Latinos are intricately connected to the future of the US (Gándara, 2010). Flores reframes the problematic of the education of Latinas/os in terms of opportunity gaps and the need to focus on the lack of access to resources that contribute to the success of privileged students. Thus, the term ‘at-risk’ should not be linked to students themselves, but to the situations that surround them and place them at risk (Gutiérrez, Baquedano-López, & Alvarez, 2000; Hill & Valenzuela, 2004; McDermott, Raley, & Seyer-Ochi, 2009; Sleeter & Grant, 2007; Waxman, 1992). These situations relate to but are not limited to a) teaching Latina/o students from either a deficiency- or a difference-oriented approach; b) consequent lower expectations affecting the quality of education through differential treatment of diverse learners; and c) blaming them for their failure based on concepts of meritocracy.

In mathematics, the NCES 2007 reports that almost half (46 %) of the Latino or Hispanic student population in 4th grade are categorized as either students with disabilities or English language learners (ELLs) and 39% as ELLs. This situation portrays the overrepresentation of Latinos in categories considered at risk. Regarding academic achievement at the same grade, the White student population is one point away from proficiency level in mathematics, and Black and Hispanic students are respectively 27 and 22 points away from proficiency. Furthermore, NCES also reports that 85% of students who are not ELLs perform at basic or higher levels in
mathematics, 42% achieve at a proficient or higher level and 6% at the advanced or higher level (See Table I). However, only 56% of ELLs perform at basic or higher level in mathematics and 13% at proficient or higher level, and no ELL performs at an advanced level. This means that 44% of ELLs perform below basic level, and only 15% of regular students perform below basic level in mathematics. This situation, combined with the fact that 40% of the Hispanic population comprises the ELL category, points to the alarming status of Latinos/as’ mathematics performance in the US. Many bilingual Latina/o students are still considered ELLs, and most of them are performing at a basic or lower level in mathematics. All students in this study are Latinas/os and bilingual. Out of the selected cases, two perform at basic or proficient levels and two perform below the basic level in mathematics. All attended a dual language program. Thus, none was considered an ELL, but the two students with higher mathematics performance also had higher levels of English proficiency.

<table>
<thead>
<tr>
<th>Levels</th>
<th>4th Grade students’ Mathematics Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ELL</td>
</tr>
<tr>
<td>≥ Basic</td>
<td>56%</td>
</tr>
<tr>
<td>≥ Proficient</td>
<td>13%</td>
</tr>
<tr>
<td>≥ Advance</td>
<td>6%</td>
</tr>
</tbody>
</table>

*Note: Below-Basic level is not reported.*

Given these circumstances, Cole (2000) argues that “[t]he difficulty is that with rare exceptions […] these populations are treated, in one way or another, as problems” (p. 374). In other words, learning and development cannot be considered apart from one’s ecological situation—including linguistic, economic, social, cultural, and political factors (Cole & Scribner, 1974; Moll, 2001; Rogoff, 1990; van Lier, 2004; Valero, 2004). This is especially crucial in the case of underrepresented groups such as Latinos (Brisk et al., 2004; Neuman & Celano, 2001; Bartolome,
In light of the foregoing, it is pertinent to focus on the social dimension of learning in the analysis of mathematics education for Latina/o students.

E. Relevance of Social Interaction and Positioning in Mathematics

In this section I describe how the focus of this study aligns with instructional and research shifts in the mathematics education agenda concerning greater emphasis on the social dimension of learning. These new perspectives view the teaching and learning of mathematics beyond isolated cognitive processes and as affected by and situated and embedded in social, cultural, and political contexts. I elaborate on these perspectives by briefly presenting the evolution of interest in this social dimension, the limitations and challenges within this perspective, current concerns and questions in the field, and research affordances of afterschool settings.

1. The Social Turn in Mathematics Education and Research

New paradigms in the US mathematics education community (and internationally) have emerged recently. These paradigms have shifted from a teacher- to a student-centered approach to instruction and from a more direct and explicit emphasis on algorithms to a more constructivist approach focused on problem solving (Chapin, O’Connor, & Anderson, 2009; Van de Walle, 2004; Woodward & Montague, 2002). In mathematics education, it is argued that “without the ability to solve problems, the usefulness and power of mathematical ideas, knowledge and skills are severely limited” (National Council of Teachers of Mathematics, 2000, p. 182). The goal is to encourage students to become mathematics problem solvers and problem posers by understanding mathematics through processes such as reasoning, representing, communicating, and proving their solutions (Lambdin, 2003; Lampert, 1990; O’Connor, 1998; Yackel, 2003). Students may not only improve the mathematical knowledge founded on their personal resources (Carraher, Carraher, &
Schliemann, 1985) but also develop productive dispositions to see themselves as doers of mathematics (Gresalfi, Martin, Hand, & Greeno, 2009; Nasir, 2002).

The related concept “mathematically powerful” (Romberg, 1994) entails processes whereby students blend together a personal sense of mathematical understandings with mathematical practices, knowledge, and skills supported through social interactions (Lave and Wenger, 1991; Rogoff, 1990). Social interactions “impact not only their [students’] mathematical thinking but also their own sense of their ability to do and persist with mathematics” (Franke, Kazemi & Battey, 2007, p. 226). Students learn and develop their mathematical identity simultaneously (George, 2009; Martin, 2007; Wenger, 1998), linking the intimate and social practices to the social environment (Arzubiaga, Artiles, King, & Harris-Murri, 2008; Holland, Lachicotte, Skinner, & Cain, 2003). It is equally critical, then, to understand who students are in a group as it is to understand who they are as individuals (Lampert, 1994) and how students construct and negotiate their learning, identities, and positions (Lee & Anderson, 2009).

The interest in the social dimension of learning mathematics began with a social turn in the focus of research within the mathematics community (Lerman, 2000). In the US it was initially influenced by the introduction of Vygotsky’s work during the late 1980s (Lave, 1988) when concepts such as participants’ goals, agency, learning processes, subjectivities, and individual trajectories became prominent and reframed school mathematics as a social practice (Lave, 1988; Watson & Winbourne, 2008). As social practice, distributed cognition (Pea, 1993), mediation during activity (Cole & Engestöm, 1993), collaborative interactions and shared meanings during participation (Newman, Griffin, & Cole, 1989) became relevant concepts. Therefore, learning mathematics has been redefined as a communal event (Radford, 2009), a cognitively self-regulated and constructive process of the learners’ situations and understandings (De Corte, 2003), and one
where issues of learning center around appropriating mathematical practices during joint activity (Brenner, 1998; Moschkovich, 2004; Schubauer-Leoni & Perret-Clermont, 1997).

Emphasis on learning as interpersonal processes that are appropriated by reconstructing and transforming them internally has generated greater interest in social participation. Thus, limiting the conception of learning to internal functioning is problematic because it ignores the situation in which it occurs. Instead, understanding learning requires focus on learners’ active stances during interactions (Rogoff, 1990). These perspectives on learning lead directly to the idea that intellectual tools are directly related to the actions and discourses of others or how students interact with others in the sociocultural environment (Moll, 2001). The relevance of interactions in the developmental process reinforces a focus on the means through which interactions occur, namely, through language. Language fulfills a communicative interpersonal function and is also an intrapersonal function for thinking and development (Vygotsky, 1978).

Consequently, my interest in positioning and participation among students derives from my assumption that learning is essentially a human activity in which interactions between student and student as well as student and adult are the roots of development (Vygotsky, 1978, Moll & Greenberg, 1990). Therefore, when interactions are positive, supportive, and unbiased, development accordingly proceeds positively. Likewise, when the interactions are not positive, supportive, or unbiased, then development can be hampered. To understand the interactions of the students in the afterschool, I examined the interactions \textit{in situ} along with the language that makes up the interactions. I expand more on the idea of limiting situations in the following section.

2. **Limitations and Breakdowns During Social Interactions**

As social interactions are infused with great power in development and learning, there is interest in the qualities of those interactions that promote successful support (Esmonde,
2009; Watson & Winbourne, 2008; Zevenbergen, 2001). However, some scholars have raised the issue of assuming that social interactions will always be productive and have suggested a need for further exploration of possible breakdowns when the interactions are not productive—or when ‘blind spots’ emerge (Moll, 2001). For example, Cheyene & Tarulli (1999) argue that, in light of how negotiations between self and others evolve and how different voices are supported, the role of dialogue in thought mediates the quality of social interactions and how students learn. Given these possible breakdowns during social interaction, I consider it important to explore this issue further, especially among student populations that are facing underperformance, such as bilingual-Latino/a students in mathematics. I believe that positioning and participation represent appropriate constructs for this study, as both emphasize the situated nature of social interactions.

The consequences of disruptions in social interactions have been studied previously through work in the fields of social psychology and sociolinguistics. This work may particularly inform the mathematics education field on factors that affect the learning process, especially of bilingual Latina/o students (Khisty, 1994). Social exclusion is one of the interruptions studied that directly relates to political issues in the larger societal context. Yet exclusion also implies its counterpart, inclusion. Inclusion refers to a group, to an associative state with others. The process of being with others and belonging to or forming part of a group also relates to the socialization process. Brewer (2005) argues that, as the need of being included in a group is met, the need to differentiate oneself from the rest arises, and lack of inclusion of an individual promotes his/her concern for assimilation. Brewer further explains that exclusion could be used as a social mechanism to punish and control others’ behaviors. One’s exclusion may be the result of others’ regulation of one’s own distinctiveness. Individuals and groups need both a collective and individual identity so that they feel that they are linked to some and separated from others and also that they are recognized as an
individual among those who are alike. As these needs are met, they may reinforce boundaries that marginalize individuals lacking a defining attribute. I consider these terms relevant to my study, as students selected groups that they wanted to be part of as they shared similar interests and also other groups that they did not want to belong to at all. Still, within the group that they selected, each of them needed to be recognized as an individual with her/his own attributes. Thus, social exclusion may permeate our lives and entails harmful emotional, cognitive, and physiological effects for those persons who are the object of exclusion (Williams, Forgas, Von Hippel, & Zardo, 2005). The positioning process relates to inclusion and/or exclusion, as some students may or may not be included at the center of the social interactions around mathematics in a group.

Furthermore, we know that inclusion and exclusion are learned processes, part of living in a social community and being someone in a given group rather than in another. Handel (1988) explains that the study of socialization is twofold: a) how a person becomes integrated in a society in face-to-face social interactions, where other agents socialize the person into way of being and doing influenced by their position in the society and b) later on how the person becomes part of different groups, and within each group there also will be other socialization agents who will take specific actions to bring the person into that new group and/or practice. These are processes through which one internalizes values and norms identifying oneself apart from and in relation to others. A person is an active agent, appropriating practices and information, participating with peers, and reproducing and extending the target culture (Corsaro, 2005). Most importantly, socialization takes place through language (Ochs & Schieffelin, 1984).

In mathematics education, Martin (2003) defines mathematics socialization as “the experiences that individuals and groups have within a variety of mathematical contexts, including school and the workplace that legitimize or inhibit meaningful participation in mathematics” (p. 16).
Mathematics socialization both reproduces inequities and promotes equity; at the same time, it implies a negotiation process between the knowledge of mathematics and the participation of subjects and their positions in that context and subject. Thus, the learning of mathematics is not only a matter of planning the content that students will have access to, but also their chances and quality of access to that content. At school, positions and exclusion are manifested through the racialized experience of minority students in the US (Apple, 1999), the dominant discourse in educational theory and practices that subordinate these groups (Solórzano, 1998; Gutiérrez, 2002), the belief of the superiority of English and the marginalization of languages other than English (Lippi-Green 1997), and the overt discrimination against minority languages in US schools, specifically against Spanish (Cashman, 2008, MacGregor-Mendoza, 1998, 2000; Trujillo, 2005).

Classroom discourse is a space of contact and confrontation of meanings (Planas, 2005). Discourse defines what is acceptable and also marginalizes values and viewpoints central to other discourses linked to societal power and hierarchy (Gee, 1990). At times these differences are manifested by the way students talk about and make fun of others and themselves (Razfar, 2006).

Therefore the phenomenon of interest in this study stems from how students’ understanding of the nature of mathematics develops through their experiences with mathematics, experiences which are made up of specific ways of thinking, doing, believing, and interacting with mathematical practices (Schoenfeld, 1994). This exploration of the quality of bilingual students’ experiences participating in mathematics comprised the main focus of the study with the purpose of finding characteristics of a social space where everyone is valued (Gee, 2005), or how the quality of interactions and the different circumstances favor or hinder their participation in mathematics problem solving. For this, I used two dialectically related constructs: *positioning* and *participation* (which I describe in Chapter II) or what Bloome, Powers, Christian, Otto, & Shuart-Faris (2005)
have described as “how participation in classroom events affects ‘who you are’ (social positioning) and how ‘who you are’ affects your participation” (p. 101). Positions are situated and discursively negotiated in interactions (Harré, 2004). Participation in a practice relates to becoming a member of the community, developing expertise and central participation in the community practices (Lave & Wenger, 1991; Wenger, 1998). It is engagement in social activity which changes the individuals and advances their ideas. With this in mind, I envision the construct of positioning as helpful in describing equitable student participation.

3. **Current Related Concerns in Mathematics Education**

This study addresses research pathways previously suggested as concerns in the field. Some of these current concerns comprise the following: a) documentation of interaction patterns that hinder or support mathematics learning (Zevenbergen, 2001); b) establishment of consequences of social exclusion in self-regulation and cognition (Baumeister & DeWall, 2005); c) understanding forms of student marginalization within interactions over time (Franke, Kazemi, & Battey, 2007); and d) description in detail of the role of tasks and ecologies supporting mathematics engagement, including ethnographic longitudinal studies of cooperative learning, and positioning processes of students in mathematics (Esmonde, 2009). We need to learn about the “hidden interaction patterns and routines that allow the participants to behave in an orderly fashion without having to keep up visible order” (Voigt, 1985, p. 81). This means that we need to deconstruct and explore the effects of what might seem natural in the mathematics educational context and realize how much these givens may be creating patterns that support only some students and exclude and undermine others. Such suggestions foresee the advancement of the mathematics education field by expanding our understandings of the social dimensions of teaching and learning in relation to student positioning (Wagner & Herbel-Eisenmann, 2009).
4. **Relevance of Research Context**

Although the data used in this study do not come from a school context but rather an afterschool one, the study of interactions during mathematics and the processes of positioning are still viable and relevant to overall issues of teaching and learning. Afterschool settings in research have proven to be resourceful environments for expanding our opportunities to observe and understand teaching and learning circumstances and students’ characteristics (e.g., social visibility or vulnerability) that in formal education settings may be hidden (McDermott & Varenne, 1998). Some examples of programs that have blended educational and research agendas include *The Fifth Dimension* (Cole & the Distributed Literacy Consortium, 2006), *La Clase Mágica* (Vásquez, 2003) and, here, *Los Rayos* (Khisty, 2004). Such contexts have allowed us to better understand the preparation of pre-service teachers for mathematics and diversity (Vomvoridi-Ivanovic, 2009) and the role of parents in mathematics (Domínguez, 2011). Afterschool settings have been studied longitudinally, but not in terms of the specific “consequences of participation for individual children” (Cole, et al, 2006, p. 13), which is the object of the present study.

F. **Preliminary Work and Research Questions**

In this section, I describe earlier studies which can be considered “pilot studies.” Previously, I conducted three related studies that are the foundation of the current study. In López Leiva (2008) I explored the reflexive and interactive positioning of Antonio, a bilingual student with learning disabilities (LD) participating in *Los Rayos*. I focused on positioning and participation constructs and found some factors that seemed to promote powerless positioning in that Antonio had little opportunity to present his own ideas and understand the task from his own perspective. Instead he had to comply. Unproductive patterns during interaction evolved when participation was ‘forced’ upon Antonio by putting pressure on him to solve the task and making personal links to his being
able to complete the task or not. Thus, side comments related to Antonio and not to his performance connected his personal identity to nonmathematical and deficit perspectives. Productive positioning patterns included those in which 1) the adult’s verbal feedback did not assign a definite ‘role’ to student’s actions but to the task and provided the student the opportunity to make choices while problem solving and 2) the interaction comprised a collective-participatory process with a collective goal and product. Another study, López Leiva & Khisty (2009), focused on teacher-student interactions. I followed a student, who we called Elsita, in two different contexts in the afterschool. In one context, she thrived in doing mathematical problem solving. In this context her preference for Spanish was valued just as much English, and there was reciprocal communication with the adult. In the other context, Elsita’s language choice was Spanish. However, English was used by everyone else, and Spanish was not valued by the group. Spanish was used to position Elsita as powerless. She was marginalized from mathematical discussions, which made her an object of microaggressions. The last study, López Leiva (2010), investigated the mathematical identities of five students. I explored reports of their experiences in two different mathematical contexts: Los Rayos and their regular classrooms. The characteristics of these different contexts appear to mediate their identity and agency differently. In the afterschool, they recognized having the support of adults and opportunities to understand the concepts in a less anxious environment with interesting, challenging, and fun tasks. The activity structures in the classrooms constrained their options, access to resources, and quality of interactions. In informal interviews, the students were able to contrast their different experiences in mathematics and in using their two languages. Their comments suggested that although they are bilingual, most of them see themselves as English speakers. Also whenever they had to pick a language to use in a task, Spanish was subordinated.
Based on the framework I have put forth regarding the nature of the social dimensions of learning and how the factors interact to affect Latinas/os’ learning of mathematics and in light of some earlier work I have done to explore some of these factors, I have come to the following research questions:

How do social interactions in small-group work mediate the positioning and participation of bilingual Latina/o students during mathematics problem-solving tasks in an afterschool program?

The related sub-questions are:

A. How does language mediate students’ positioning and participation?
B. What are the emergent patterns of positioning and participation over time?
C. How do students make sense of their positioning and participation in mathematics?

G. Summary

Essentially in this chapter I described the emergence of my interest in the phenomena of positioning and participation. I supported how these research constructs aligned with current, relevant concerns in the mathematics education field. This perspective especially views learning and participation dialectically related; thus the emphasis on student active collaboration in groupwork has increased. Positioning is a helpful construct for examining the quality of the social interactions during collaborative processes, as it highlights who has more opportunities as well as the quality of those opportunities to engage in and learn mathematics. Therefore, the pursuit of the evolution of this construct represents a meaningful area to explore, understand, and learn, particularly in the case of supporting learning ecologies of bilingual Latina/o students or other underrepresented groups of students learning mathematics. In the following chapter I present a review of previous studies that inform the present study.
II. REVIEW OF RELATED LITERATURE

A. Overview

This review explores the literature of related theoretical definitions of the main constructs in this study i.e., positioning and participation. Next I elaborate on two special studies directly related to this study. Then I present a collection of empirical works developed on positioning in the areas of mathematics and science education, social interactions in mathematics, and social interactions and exclusion.

B. Positioning

In this section I introduce a general definition of positioning. Next I present different perspectives on positioning, which respectively present alternative units of analysis, and I expand on the different kinds of analyses developed in light of these definitions of positioning.

*Expectation States Theory* (Berger, Cohen, & Zelditch, 1972; Cohen, 1984, 1994, 2000) is an approach closely connected to positioning. This theory explains how status, which is socially constructed and contextually situated, becomes a basis for expected competence in collective tasks. Status transfers across settings as people identify and reinforce it even when the social status might not be directly related to the task at hand. Individuals with higher social status are assumed to be more competent (Cohen, 1984). So people with high social status are more likely to look for or find similar positions in every context in which they participate. Academic performance, race, gender, and other similar factors serve as mediators for the reification of status levels. In fact, in a study of peer interaction in a bilingual mathematics classroom, Cohen found that children who were not proficient in English were perceived by their peers as not being good in mathematics and reading, and those fully proficient speakers, with higher social status, had greater rates of participation. “If children can […] understand that there is not just one ability that is relevant to new learning tasks,
but a number of unrelated activities, then the expectations on […] preexisting status […] will be weakened as they combine […] expectations based on multiple abilities” (Cohen, 1984, p. 184).

The related term positioning has been defined and studied in various ways. The first one that I explain was developed by McDermott, Gospodinoff, & Aron (1978) who argue that positioning refers to physical postures along with discourse that differentiate students’ responses during joint social activity. Similarly, Rogoff (1990) used positioning to denote physical postures or locations where mothers placed their children and used them as means of communication.

Other perspectives take more social and psychological approaches. All of them argue that communicative processes comprise the major means of positioning (Goffman, 1981). They also argue that positioning evolves in the kinds of participation of a person in a community, having roles that are marginal, peripheral, full, and/or central (Wenger, 1998). Furthermore, others argue that positioning is a situated, culturally and historically mediated process that affects experiences and the subjectivities of the person. Persons’ subjectivities are not necessarily connected to institutions, but to the social processes organized in the practices in those spaces, processes which give meaning to those practices and at the same time determine the positions of members in relation to others. The repetition of positioning as it evolves in everyday interaction informs our identity, and over time it works as a lamination process (Holland & Leander, 2004). Positioning happens across time and contexts through language, interactions, objects, goals, etc. (Holland, Lachiotte, Skinner, & Cain, 2001). In education, members’ positions develop in relation to those of others (Walshaw, 2005). Social positioning in interactions comprises a subtle and dynamic co-construction among subjects that also influence the social structures of institutions. Some categories give subjects their social identity (e.g., race, gender, class, and ethnicity), some are given by the institution, and some are constructed within conversations. Some may be resisted and others accepted and adopted. These
social identities may or may not affect how participants interact, but how they interact with each other contributes to their identities. Social status, privileges, and educational opportunities as well as marginalization, removal, and denial of personhood status are linked to institutional identities (Bloome, Powers, Christian, Otto, & Shuart-Faris, 2005). In mathematics education, Nunes (1999) describes the alienation of some learners by excluding their forms of knowledge, by a “selective ignorance” that teachers acquire growing up by participating in specific cultural practices that lead them to think that some things are to be known and others ignored, depending on one’s social position.

*Positioning Theory* (PT) interprets social interactions between people by examining how their different status and power relations intersect in a specific context or activity (Ritchie, 2002). More specifically, PT (Davies & Harré, 1990; Harré, 2004; Harré & Van Langenhove, 1999; Harré & Moghaddam, 2003) studies the nature, formation, influence, and ways of change of local systems of rights and duties shared in small-scale interactions. Harré & Van Langenhove (1999) argue that the “social recognition […] of a given persona will have profound effects upon the ways in which the person’s behaviour is viewed and the ways in which the person is then treated by others” (p.8). Harré (2004) describes positioning as a discursive practice, as a construction of personal stories in the assignment of “roles.” Thus, it is the practice of rendering a person’s actions as intelligible social acts. Positioning (defining someone as powerful or powerless, confident or apologetic, dominant or submissive, definitive or tentative, etc.) can evolve in two forms within a text or interaction: reflexive positioning (i.e., a person positions him/herself, e.g., through the use of pronouns) and interactive positioning (i.e., the person is positioned by others). People interact as a kind of participant, and they create their story, simultaneously affecting and being affected by those around them, who, in turn, co-produce their storylines. Positioning evolves in four different forms:
a) deliberate self-positioning, b) forced self-positioning, c) deliberate positioning of others, and d) forced positioning of others (Van Langenhove & Harré, 1999).

Harré (2004) describes the conditions that determine the flow of the positioning during the interactions: a) the set of admissible social acts and meanings in what is said and done; b) the distribution of rights and duties, each distributing a position; and c) storylines, defined by their origins and plots during communicative processes or interactions within an episode and/or previous ones. Conditions are dynamic, often contested and contingent structures, and people differ in their willingness to initiate storylines. All conditions determine one another. Ideas about rules mediate the meanings of “speaking and acting, and both are influenced by and influence the storyline.” Episodes unfold depending on how any of these three aspects is directed and challenged.

Storylines are formed through collaborative narratives developed on knowledge of cultural structures and the positions familiar to the participants, which participants constantly refer to in order to position themselves and others through discursive actions. Perspectives of storylines vary from one to another because these may be understood differently by different people and some may contradict themselves. As individuals participate in social interactions, they are constituted and reconstituted through the various discursive practices, so one’s identity is not fixed, but fluid and often contradictory (Davies & Harré, 1999; Harré & Gillett, 1994; Ritchie, 2002). Stories evolve in different discourses and vary in terms of language used, issues, and values, and positions emerge with them (Davies & Harré, 1999). This analytical process demands researchers to suspend their theories of what is going on and look at what people are doing in their context (Harré, 1995).

Furthermore, Bamberg (2008) expands the notion of PT to positioning analysis, which he defines as a type of micro-ethnographic analysis within psychology. He contests the idea that PT has focused on the “who-is-in-charge” dilemma, which in his view contradicts the agentive
organism and emphasizes more strength in societal structures. Conversely, positioning analysis studies how people as agentive actors position themselves and how, in doing so, they become positioned. The analysis first starts with the subject’s actions and then continues with a sequence of interactions within the context of others’ actions, though the subject’s identity is still maintained, thus creating a simultaneous integration and differentiation of the personal process to the collective one. Bamberg defines this as a moment-to-moment functional analysis process where one can analyze the subjects in a time and a space, but also how the “world” is constructed around the needs of the subjects themselves or how the teller wants to be understood.

While a macroanalytic approach would start with a concept of the subject as primarily socially constructed by the outside, Bamberg proposes starting from a micro-situated formation of identities and otherness focusing on the practices in which the subjects evoke dominant discourses or master narratives, then linking them to the social world (i.e., the macro-level). In mathematics education, positioning has been described as a twofold process of the interaction between the power relations in the context and students’ actions. Positioning evolves through the expectations of students’ actions in relation to the distribution of authority in the context of interactions and by the ways students exercise their agency (Gresalfi & Cobb, 2006).

More specifically to PT in mathematics, Wagner & Herbel-Eisenmann (2009) developed a series of metaphors to explore positioning. They claim that changing the way mathematics is talked about will in turn change how mathematics is done and taught. Their metaphors comprise images describing the immanence, reciprocity, contingency, and contestability of defining positioning. Bernstein (2000) describes education as a recontextualization process of ‘real’ out-of-school practices through similar power and control structures which generate different positions during classroom relations. The inherent school structure and processes are often permeated by dominant
groups’ perspectives and social ideologies that in turn determine school practices and produce the social positions of teachers and students. Thus, in mathematical practices specific positions evolve through similar procedures. For example, in one pair a student may be positioned as more able and is expected to help another. This situation is not indexed by the mathematical content of the task, but how students are positioned during their interactions in joint activity (Lerman, 2001).

Parallel to the previous stances, Esmonde (2009) argues that “classroom activities, like mathematical talk, do more than just convey mathematical ideas. Every moment of classroom interaction is also a part of constructing, challenging, and reifying students’ social positions and identities—some of which are constructed primarily in the classroom, some of which are related to the broader communities in which students participate” (p. 1112).

C. Participation

In this section, I describe how participation and learning have been considered parallel processes. I expand this perspective by describing participation in mathematics and the relation of participation with positioning through the participation framework. Learning emerges through participation. Learning is a situated process that normally occurs as a function of participating in an activity (Lave & Wenger, 1991). The process moves learners from “legitimate peripheral participation” to a central role with an increasing load of responsibility and knowledge in a “community of practice” where experts, through social interaction and collaboration, promote beliefs, practices, and behaviors valued in that social space. These participation trajectories change the division of labor and participation patterns, thus altering the community’s ongoing relations. These changes, which are linked to power relations, emerge during their negotiation and thus
constrain participation. Rogoff (1994) asserts that learning implies a transformation\(^1\) of participation itself, through the intersection of interpersonal and cultural surroundings and through the individual’s effort (2003). Participation transforms the context and the self simultaneously through social activity (1990). Thus, participation in collaborative processes in mathematical practices transcend to the transformation of what is appropriated, to the creation of tools and operations that mediate the reconstruction of the practices observed (Cole 2006; Engerström, 1999; Moll, 2001; Roth & Lee, 2007; Wells, 2002).

Participation is a process that evolves in continuously renewed social structures or relations, thus bridging experience and understanding. In mathematics, participation could be portrayed as having access to the ongoing activity, the practice of mathematics (Lave, 1988; Wenger, 1998; Resnick, Bill, Lesgold, & Leer, 1991). Therefore, during interactions it is not enough to note that some students participate less centrally than others, but rather how to support students in peripheral places to gradually move themselves into a more central position or at the same time detect whether their participation might be truly marginal. This implies exploring all forms of student participation to acknowledge the opportunities when they access more central participation (Esmonde, 2009). As students participate in mathematics, especially in problem solving, issues of collaboration may emerge. Pea (1993) argues that problem solving should be a distributed product of collective intelligence activity, where participants, as part of a participatory-reflective process, draw from and apply their own and others’ resources as tools for learning. Thus, students’ competence is not situated in themselves only, but in social relationships and artifacts in the context where they problem solve (Lave, 1988; Kirshner & Whitson, 1997). In mathematics, problem-solving performance is viewed as the result of interconnected elements: a) *Resources* refers to what problem

\(^1\) Learning can evolve in three models: transmission (knowledge is passed from knowers to learners), acquisition (the learner constructs it independently), and participation (the learner builds it in a community).
solvers understand and bring with them to tackle the problems, b) Heuristics refers to the techniques they use to progress through the challenging situations, c) Control refers to managing the information and resources in the implementation of the solution plan, and d) Beliefs refers to one’s mathematical worldview (Schoenfeld, 1985). So, as students participate in problem solving, they make use of and share these elements.

The nature or quality of participation during interactions has been the concern of several authors, especially Goffman (Levinson, 1988; Giddens, 1988). He was interested in the micro-analysis of face-to-face interactions in events, in the co-presence of individuals “seek[ing] to understand what goes on in terms of their participation within a particular form of encounter, not in terms of their membership of the group […] but in the ‘interaction order’ […] where tensions emerge” (Giddens, 1988, p. 257). Goffman’s participant framework comprises the participant structures (rights and responsibilities of interactions in particular activities) and speakers’ depictions of others or the production and reception of roles in situated activity and their connections to communication, power, time, space, and self-image in relation to micro- and macro-structures (Levinson, 1988; Giddens, 1988). Goffman (1981) called “animation” the way a speaker animates self or other as a figure by linguistic means. Through talk, speakers give each other roles and social identities relevant to the moment, whereby they align with or oppose each other and at the same time position each other with respect to topics and utterances. Thus, as students participate and learn, they take on different positions or roles (Harré 2004; Holland & Leander, 2004; Radinsky, 2008). These aspects dialectically link participation with the positioning process.

D. Studies on the Social Dimension of Learning and Positioning

Before providing a summary of the related works in these areas, I would like to introduce two specific pieces that I consider especially relevant to my research interests. First, Schubauer-
Leoni & Perret-Clermont (1997) describe their 15-year research trajectory, moving from a purely cognitive stance to a socio-cognitive one. In their description they explain their modification of questions and methods according to their results regarding the roles of social factors in development, more specifically, in mathematics learning. They started with a Piagetian task—i.e., from a constructivist approach more cognitively and individually than socially driven—of additive problems \((a + b - c = X)\) in a context with sweets and an empty non-transparent bag. Results verified children’s situated mathematical experience, as they did not use arithmetic writing.

Therefore, the next stage included four conditions, either one or two peers working on the same task, and they prepared their findings to explain them to another child. Finally, an additional factor was whether this last participant provided feedback to the original group or not. Results supported that students who were in pairs and had feedback made the most progress (93%), whereas 76% of the children working alone and with no feedback showed regression. This underscored the notion that mathematics understanding is not the product of the individual mind of an isolated child, but a socio-cultural practice made accessible to the child through social transmission. Progressively, this study included dimensions, such as functions of mathematical tasks, problem-solving strategies linked to their previous experiences, adult presence and the construction of knowledge, and the interconnection of all these elements. Thus researchers moved from an outcome-based to a process-based approach, an ecological system of relationships. And they found teachers’ differential treatment of students from privileged social backgrounds, power-dynamics in peer relations, and institutionalized influence of school and out-of-school contexts in students’ mathematics thinking and performance.

Second, McDermott & Varenne (1998) describe the trajectory of a student, Adam, in four different settings (everyday life, afterschool club, classroom groups, one-on-one tests) and how his
differences (e.g., difficulty to read) became institutionally marked and how he was “acquired” by a learning disability. The authors argue that although there might have been something different about Adam, the difference was not a source of the practical problems he encountered at times in his life, but the well-organized context—with cultural and institutional practices—that labeled and disabled him. The authors claim that “take away the institutions or limit their sphere of relevance and the ‘problem’ disappears even if the difference does not” (p. 42). Although Adam developed sensitivity to his factual conditions, his resistance to the pain of the process did not affect his career. In similar situations other students either developed defiant behaviors or dropped out. Also, McDermott, Goldman, & Varenne (2006) mentioned three patterns in special education: a higher percentage of minority children than of White children are assigned to special education; White children are assigned to less restrictive programs than minority students; and the data—driven by inconsistent methods of diagnosis, treatment, and funding—make the system difficult to describe and change.

I believe these two examples, from Leoni & Perret-Clermont (1997) and McDermott & Varenne (1998), support my claim that the social dimensions of learning are not only relevant in learning mathematics, but they, at times, may also become limiting.

E. **Review of Related Empirical Research Literature**

I present here current studies that consider the social dimension of learning, especially positioning and participation, or social interactions in the areas of mathematics or science education.

1. **Positioning in Mathematics and Science**

   This part is divided into two subsections. The first includes studies that specifically developed their analysis using PT. Two of these studies were conducted in science classrooms, and another included English Language Learners (ELLs). The rest are in mathematics education. In the second subsection the six studies explored positioning but not as a central construct.
For the first subsection of studies using PT, I present three studies exploring teacher-student interactions mediating positioning. Black (2004) investigated effective teacher-student talk by using a multi-layered approach to the analysis of whole-class discussions through video and audio data, collected over a five-month period in a primary school mathematics classroom. Black highlights that teacher’s expectations and time pressure influence the quality of interactions students experience, and thus also their social positioning, which simultaneously affects students’ access to learning processes. Similarly, Yoon (2008) examined classroom teachers’ views of their roles regarding ELLs and their relation to their teaching approaches, the students’ reactions, and positioning in the classroom. Results showed teachers’ approaches relate to ELLs’ participation and self-positioning as either powerful or powerless students. Likewise, other students followed the teachers’ model in their interaction with ELLs. As a result, teachers’ active or passive actions marked ELLs as acceptable or not. Yoon concluded that student-centered approaches may not be useful to ELLs in contexts with hidden power relations unintentionally isolating ELLs and positioning them as powerless. Finally, Wagner & Herbel-Eisenmann (2008) analyzed a series of middle schools’ classroom discourse, observing the frequency of words and how positioning is encoded in language practice. They noted that roles given to students emerged in relation to the storylines developed by the interactants and that various positions may emerge in a storyline. In the process, teachers made subtle uses of language either to invite or repress student dialogue or initiative. Thus teachers reiterated their authority through the classroom discourse format. These studies emphasize teachers’ influence on student positioning.

Two other studies explored positioning constructed among students. Ritchie (2002) found that in 6th grade science activities one female dyad of students had an unproductive journey as they interacted with students in both mixed-gender and same-gender groups. He noticed how gender,
status, and power relations intersected during opportunities for science learning. Dunleavy (2011) analyzed high school students’ perceptions about who is considered smart in mathematics. Students assigned powerful positions to other students who either explained or answered more questions during class than the rest or were in higher school grades. Additionally, they argued that while working in small groups, students’ status was affected through imbalances in their performance (i.e., difficulties collaborating or feeling less competent), and their positions were stronger when they collaborated and had equalized participation. Thus, what students co-construct as more relevant during interactions seems to serve as a compass during their interactions.

In the last study exploring how ideologies shape interactions, Evans (2000) describes how metanarratives dominate the classroom and the society, distributing powerful and powerless positions to subjects. Positioning mediates mathematical thinking and the way individuals identify themselves in relation to mathematics. Evans analyzed interview responses of adults, focusing on the social or transcendent discourses and positions that these adults took as they were interviewed. He noticed that subjects took multiple positions as they spoke depending on their relation to a topic.

The second subsection of studies considers positioning, but not as a main focus. González, Andrade, Civil, and Moll (2001) argue that the acknowledgement of the funds of knowledge in a group or classroom mediate different positions for the participants in mathematics, thus indexing that privileging and marginalizing processes are socially constructed and, as such, are discrepancies that could be bridged by creating zones of mathematical practice. Nasir & Saxe (2003) also argue that classrooms provide different sources of cultural capital that mediate how students may perceive their places or positions in the interactions. Focusing on communication, Sfard (2001) identified how the discipline’s discourse shapes the interactions, mutual positions, and identities among the participants through discursive moves. Lerman (2001) also explored how practices of mathematics
produce different positions for students as they interact. Sometimes it is not the content but how students are supported or instructed to work, thus distributing positions among students that they can take up or resist (e.g., who leads, who talks, who describes). In this process the students’ narratives are helpful to understand their perspectives about the events in which they engaged. This analysis, however, seems limiting in exploring students’ interactions across time. Sensevy, Schubauer-Leoni, Mercier, Ligozat, & Perrot (2005) argue that positions are fluid and positioning changes. They described that as teachers take actions or assume certain positions connected to authority, they make the classroom’s positions obvious to everyone. For example, the teacher may lead the lesson, telling students what to do (using nouns), but positions become more flexible when teachers act at the same level as the students (using verbs). The latter allows teachers to position themselves in more open terms. Finally, using Goffmans’ participation framework, Empson (2003) explored the successful experiences of two low-performing 1st grade students in mathematics. She argues that despite their limitations in the content area, these students became successful given the social and strategic support provided in class by building off of students’ previous understandings, positioning them as mathematically competent continuously, and providing interactions that reinforced their identities.

In conclusion, this combined set of reports describes the elements that affect the positioning process: the teacher’s mediation; the development of what becomes “relevant” during interactions; the connection to culture, ideologies, and metanarratives; the use of discourse; the distribution of work; and the consideration of and capitalization on students’ previous knowledge. All these elements, at the same time, support the conditions suggested by PT linked to the structure and function of social interactions that mediate the quality of opportunities for the participants.
2. **Social Interactions in Mathematics**

In this section, I present twelve research studies that explore the role of interactions in mathematics during collaboration involving the teacher, peers, and/or technology; deficit ideologies infiltrating the classroom; and relations between mathematical identities and abstractions.

On collaborative interaction, Blair (2004) studied secondary school students’ difficulty in selecting appropriate heuristics during problem solving and how their self-regulation resulted from social interaction. Therefore, Blair found that interactions might better support how students engage in problem solving. Further, Schliemann (2002) argues that meaningful learning is promoted not only in social interaction but also in each student’s perspective, reasoning, and construction processes. Likewise, based on results from a longitudinal case study in an elementary school, Askew (2008) argues that mathematical understanding and learners’ social identities are inseparable. Identities are situated and develop within the affordances and constraints of the classroom culture. Lastly, using 3 student cases engaged in mathematical activities, Lerman (2009) demonstrates that mathematical learning is more than just knowledge. It is an inductive process into the culture of mathematics (skills and conventions) connected to the emotional charge of previous experiences. These results show that when students do mathematics, they are not doing it only with focused minds, but they are holistically participating in the process. Learning mathematics is not only knowing but also feeling confident about doing the tasks. Thus, a culture that welcomes students into learning mathematics is crucial for the expected outcomes.

On teacher-student interaction, Lau, Singh, & Hwa (2009) use the zone of proximal development as a framework to explore the discourse in videotaped lessons and teacher and student interviews from a mathematics classroom over a 10-month period. They found that students developed different types of interactions in mathematical sense-making and justification of ideas.
and arguments through interactions with the teacher. Exploring continuities between teachers’ and students’ experiences of school mathematics, Hodgen & Marks (2009) found that the quality of relations in mathematics reproduces the negative and stigmatizing conceptions of ability. The teachers often reproduce the same pattern of inclusion and exclusion to which they were subjected. They suggest an assessment that focuses on listening to students and promotes learning rather than labeling. The process of replicating interactions seems relevant to this study.

A relevant study for my research is one in which Gorgorió & de Abreu (2009) explored teachers’ representations of immigrant students at the macrolevel and at the microgenetic level of ideologies. They found a need to critically revise teachers’ perspectives on diversity, as they seemed heavily influenced by unchallenged deficit perspectives. This study shows how the previous ideas or dispositions that teachers have may affect how they see and interact with their students. In a yearlong ethnographic study of one middle- and one working-class classroom, Zevenbergen (2001) found teachers’ use of triadic dialogue in mathematics classrooms: the teacher questions, the students respond, and the teacher evaluates the response (IRE). IRE involves many students but in low-quality interactions. While middle-class students complied with the IRE structure, the working-class students either resisted or failed to recognize the structure. These situations favored some students and excluded others in active, but subtle and coercive ways through mathematical practices and discontinuities between students’ linguistic habits and those of the classrooms. Zevenbergen (2000) argues that students need to learn how to “crack the code” of mathematics by understanding the patterns of language, work, and power implicated in the construction of mathematics. This calls for a change in practices by not watering down the mathematics, but rather making these accessible to students by explicitly targeting the embedded mathematical practices.
In relation to using technology, Zurita & Nussbaum (2007) and Li (2004) declare the need for collaborative group activities that promote student social interaction. Based on Activity Theory, with 24 6- and 7-year-old children in a month-long study, the former present a method and framework for a computer-supported collaborative learning system using wirelessly interconnected handhelds. This system created new opportunities and positive effects for motivation, social interaction, and learning, thereby changing classroom pedagogical practices. The latter, also based on a context of computer-mediated communication, indicated that students actively collaborated by using two language functions: giving explanation and expressing disbelief.

On mathematical abstractions, Ozmantar & Monaghan (2008) observed two teenage girls working together and argue that abstractions develop from situated needs and from the context. They do not evolve from concrete to abstract, but in a dialectical relation between the concrete and the abstract. Therefore, while students’ perceptions may seem relevant at the moment of engaging with each other and in a task, interactions are not comprised of the present situations only. They extend to other experiences that affect one’s interpretation of the current situation. Ozmantar & Monaghan argue that the situation involves “who you are, where you are from, what you know, what you respect, whom you get on with, how you get on with them, what you are doing, why you are doing it, what you are doing it with, [and] how it is structured” (p.125).

This set of studies on social interactions also reinforces what studies on positioning address, namely that the quality of the collaborative process mediates how students access mathematics. In this process the quality of external factors (teacher, peers, structures of interaction, and ideologies) merging with internal factors (students’ emotions, understanding of the context, resources, and identities) evolves depending on a genuine, situated accommodation with one another, away from stereotypes.
3. **Social Interactions and Exclusion**

In this last section I discuss seven studies that explore breakdowns in social interactions. In special education, Frederickson & Furnham (2004) found that typically achieving middle school students rejected their peers with learning disabilities (LD) for both play and academic activities despite the inclusive setting. Plata, Trusty, and Glasgow (2005) showed similar patterns in connection to race and ability. In fact, in a meta-analysis of social status among children with LD, Kavale and Forness (1996) found that 80% of students with LD self-report having low social competence, perhaps in connection to the social rejection exercised by their typically achieving peers. Studies on exclusion demonstrate that groups often develop exclusionary practices as a means to maintain their collective identity. This process facilitates and supports stereotyping, which in turn shapes social judgment and behavior and promotes preferential attitudes that reify characteristics central to the dominant group (Abrahams, De Moura, Hutchinson, & Viki, 2005; Hogg, Fielding, & Darley, 2005; Gaertner & Luzzini, 2005). Finally, Mazzei (2003) is a study on silencing that presents gaps in research areas or topics often connected to issues of race and culture. She proposes listening to the silences. One way would be to explore the experiences of the inhabitants of silence, to notice present and absent meanings, their sources, and motivation. In conclusion, this section supports that stereotypes, especially those regarding perceived ability and race, are prevalent and hinder social interactions.

F. **Summary**

Although the bulk of consulted works originated from different fields, they all agree on the deep connection of how social aspects, such as interactions, opportunities to participate and self-perspectives about the task at hand and oneself, are crucial in the process of learning and doing mathematics. Power embedded in discursive actions affect the evolution of social interactions.
III. THEORETICAL FRAME

This study centers on critical issues of equity in mathematics education. Rather than “gap gazing,” it explores the “mirrors and windows,” or students’ opportunities, access to resources, and cultural and linguistic identities (Gutiérrez, 2008). In the exploration of bilingual Latino/a students’ opportunities to engage in the meaning-making process and social interactions during mathematics problem solving, I considered it relevant that the study be founded on a sociocultural perspective. This perspective posits the immanently social nature of learning. Therefore, to explore social interactions I used Vygotsky’s (1978) concept of the zone of proximal development (ZPD) as an underlying basis for meaning-making processes, where transformations at the external interpersonal level are transferred to the intrapersonal level and mediated through language and the resources accessed by the participants during interactions. The ZPD in the afterschool was generated through small communities of interaction comprised of students, facilitators, and, at times, mothers. The quality of interactions varied depending on the how participants came together in a context in which they could have mutual trust and access bilingualism and different types of discourse as needed (Moll & Whitmore, 1993). This was especially true in that they had access to equalized interactions and shared sense making and in that personal resources were enhanced (González, Andrade, Civil, & Moll, 2001). At times, the quality of ZPD was affected by how interactions were co-constructed (Daniels, 2001; Moll 2001) around power issues.

Discourse is a social space where power is contested and negotiated (Bloome et al., 2005; Fairclough, 1989, 2003; Gee, 1990; Planas, 2005). Fairclough further argues that control over orders of discourse is a powerful mechanism for sustaining power. Therefore, powerful participants can exercise control over the contributions of non-powerful participants through constraints on what is said or done (content), social relations people enter into during discourse (relations), and the
subject positions people can occupy (subjects). These processes of power might relate to immediate and concrete power exercised during discourse or also through structural power often indexed through ideologies or generalized social practices. This becomes especially true in mathematics discourse and interactions (Valero, 2004; Martin, 2006; Wagner & Herbel-Eisenmann, 2009).

Based on the possible differences that can develop during interactions, I focused on two main constructs in this study. The first one, *participation*—considered a fundamental process in learning (Lave & Wenger, 1991; Rogoff, 2003)—refers to how individuals socially engage in a task. I used Goffman’s (1981) participant framework, especially in the exploration of the quality of experiences by analyzing the participation structures (rights and responsibilities), that is, how participants are “animated” or assigned roles as they interact. The second construct, *positioning*, intricately linked to participation, refers to how individuals are perceived by others and how they present themselves in relation to one another during an activity. I used positioning theory to explore this construct (Bamberg, 2008; Davies & Harré, 1990; Harré, 2004; Harré & Van Langenhove, 1999; Harré & Moghaddam, 2003; Holland & Leander, 2004). Positioning unfolds within the presence and negotiation of three conditions: *speech acts, storylines, and positions or roles* (Harré, 2004). The processes begin with how persons locate themselves and others in the social spaces using categories and storylines. Then these ideas are negotiated, and finally they can be reflected on or noticed by the self. This process may evolve either deliberately or forced by oneself or others (Van Langenhove & Harré, 1999).

To interpret interactive processes through language, I drew on the ecological perspective of language (van Lier, 2004). This perspective emphasizes the importance of the social, complex, and situated nature of interactions in the environment where they evolve, thus co-constructing one another reciprocally. I used three concepts of relations from this perspective. *Emergence* refers to
how simple elements reorganize themselves into more complex systems in adapting to changing conditions. *Affordances* refers to possibilities for action and opportunities for engagement and participation which stimulate the development of joint interaction that furthers actions in a context. Affordances create relations between subjects and their environment; orientation toward the production and recognition of structural patterns; demands and requirements, opportunities, limitations, rejections, and invitations in the linguistic world; and whatever is available to the person to do something with. Affordances are not neutral. *Noticing* refers to how participants establish a relation between the resources that are available to subjects in the activity at hand and the meaning that they might take in the environment where the active engagement takes place. Thus a connection between perception/activity and environment/agent is created.

I integrate all the above perspectives: the participation framework, positioning, ZPD, power, and the ecological perspective in two models which summarize my theoretical frame. The first model (Figure 1) was adapted from the ecological perspective (van Lier, 2004, p. 183). This portrays three lines of power that affect and structure interactions. Each line represents a continuum of power in the areas of bilingualism, collaboration and mathematical practices, respectively:

![Figure 1: Lines of power structuring and affecting interactions](image)

The midpoints in these continua represent a more balance distribution of power, and the ends represent the practices with contrasting levels of power in each category. On the *bilingualism* continuum, English in a written form is considered more inherently at a higher level of power in the US context, as opposed to spoken Spanish. Similarly, on the *collaboration* continuum, the self
isolated without social uptake during participation represents less power, as opposed to a group working only amongst themselves, ignoring the self and/or a member in the group dominating the interaction. Finally, on the mathematical practices continuum, the formal, academic discourse and standardized procedures in mathematics index higher levels of mathematical power, as opposed to informal ones.

In the second model (Figure 2), I integrate the immediate interactions constructed through participation, learning, and positioning with processes such as identity and socialization that develop over time. Thus, for the longitudinal exploration of the constructs of this study, I used the concept language socialization to analyze accumulated levels of positioning and participation over time (Brewer, 2005; Handel, 1988; Jackson, 2009; Lerman, 2009; Ochs & Schieffelin, 1984), as well as student mathematical (George, 2009; Martin, 2006) and bilingual identity (Cashman, 2008).

**Figure 2**: Immediate and over time processes in social interactions around problem solving in mathematics.

Although the research supporting this theoretical framework was presented in the previous chapters, there are still some terms related to this study that I would like to define here.

*Mathematical identity* entails “the dispositions and deeply held beliefs that individuals develop
about their ability to participate and perform effectively in mathematical contexts, within their overall self concepts” (Martin, 2006). Any change implies a change in position and status of a person (Martin, in press). Bilingual identity denotes recollections of experiences and beliefs about two languages from intimate and socialized events relating to language skills, a sense of value regarding one’s linguistic preference (in this study English and/or Spanish). Language socialization refers to the fact that the socialization process emerges through language and also the process in which we are socialized into language (Ochs & Schieffelin, 1984, 2008; Jackson, 2009). Mathematics socialization designates “the experiences that individuals and groups have within a variety of mathematical contexts, including school and the workplace, and that legitimize or inhibit meaningful participation in mathematics” (Martin, 2003, p. 16). Mathematical discourse practices refers not only to meanings of utterances, ways of talking, acting, interacting, thinking, believing, reading, and writing, but also to goals, mathematical values, beliefs, points of view, and the focus of attention (Moschkovich, 2003). Language and meanings are not mathematical in themselves but in that they are embedded in mathematical practices. This implies “talking and acting in the ways that mathematically competent people talk and act. These practices involve much more than the use of technical language” (Moschkovich, 2002, p. 199).

In essence, this theoretical framework addresses how the role of discourse during interaction is pivotal for communicating, thinking, and positioning one’s and others’ roles, actions, and ideas in mathematics. Social processes function as bridges and nurturing elements among people; yet, in the light of socially constructed power, they also discriminate and work as a sorting mechanism of the kind of connections to be established, with whom, and how. Thus, it is not power itself which links or separates people, but it is their coming together in a social space that generates the power, which is at the same time used, negotiated, and applied. I envision these three lines of power (bilingualism,
collaboration, and mathematical practices) creating a three-dimensional Cartesian coordinate system that could represent how the participants’ social and mathematical positions co-constructed during interaction would fall at different points on these planes respectively, mediating different levels of mathematics and social performance for the participants. This representation would illustrate how, despite the physical proximity of the participants and besides the sharing of the same social space, their circumstances would not be exactly the same.

The purpose of analyzing the phenomena of positioning and participation within a theoretical frame like this, as Lantolf (2000) states, is to explore how this process functions through its conditions and contextualized nature in the environment where the activity takes place. Human activity starts motivated by needs; needs are realized into motives once they are directed to a goal:

“Motives are only realized in specific actions that are goal directed (hence, intentional and meaningful) and carried out under particular spatial and temporal conditions (or what are also referred to as operations) and through appropriate mediational means. […] Activities then can only be directly observed, by others, at the level of conditions. However, the motives and goals of particular activities cannot be determined solely from the level of concrete doing, since the same observable activity can be linked to different goals and motives and different concrete activities can be linked to the same motives and goals” (Lantolf, 2000, p. 8).

I further analyzed mathematics problem solving using activity theory (Engeström, 1999), thus envisioning mathematics problem solving as an activity. I especially used the notion of contradictions between or within activity components. I see in this notion related to the construct of positioning as it describes imbalances that develop in an activity regarding the object and subjects of the activity, its goal, its means, its community, and the distribution of labor among participants. Therefore the goal in this study is to explore the operations that promote the positioning and participation of bilingual students through social interactions around mathematics and to explore their own perspectives of about becoming bilingual doers of mathematics. In the next chapter I describe the methodology utilized in this study.
IV. METHODOLOGY

A. Overview

In this chapter, I present this study’s research questions, context, and the rationale for its research design (i.e., case study) which are based on sociocultural and sociopolitical perspectives. I describe the relevance and appropriateness of approaches included in this study, such as its methodology, the type of case study, the selection of participants, and the sources of evidence. Additionally, I also present the plan, strategies, and tools utilized in the process of organizing and analyzing the data.

B. Research Questions

Given this study’s concerns regarding the quality of social interactions in mathematics and how these affect participation, its setting, and participants, the overarching question in this study is:

*How do social interactions in small-group work mediate the positioning and participation of bilingual Latina/o students during mathematics problem-solving tasks in an afterschool program?*

The following are related sub-questions:

- How does language mediate students’ positioning and participation?
- What are the emergent patterns of positioning and participation over time?
- How do students make sense of their positioning and participation in mathematics?

C. Why Qualitative Methods and a Case Study

My interest in the social constructs of positioning and participation as they were co-constructed through social interactions of the participating students led me to use a qualitative approach because I was interested in a thorough description of how these processes evolve. This exploration is a sense-making process of the phenomenon, not with the purpose of explaining what is happening, but with the purpose of trying to understand the phenomenon and its meaning to the participants (Erickson, 2005; Greene, 1997). The context of this study included several (twelve) students who participated during the entire program. I wanted to explore both a) how the
interactions of the students and their use of language mediated their different positions and participations as they worked in groups while solving mathematical problems and also b) how their positions and participation in mathematics changed or not over time during their attendance to Los Rayos. Therefore, I decided to use a case study approach by selecting only some students and focusing on their participation in small groups. The reason for exploring the participation of these students when they were in the same group was to portray the kinds of affordances that interactional processes can provide to different members within the same group.

Case studies usually analyze a system of actions rather than an individual or group of individuals (Yin, 2009). With the purpose of better understanding how social interactions mediate the positioning and participation of specific students in situated exchanges and over time, I used a case study design with an ethnographic emphasis. In addition, this ethnography has a participant observer element since I was an active member of the afterschool and took regular fieldnotes.

A case study is an in-depth holistic analysis. It is one that examines the situations, circumstances, and/or subjects with the purpose of understanding these elements in real-life contexts, without altering them (Stake, 1995). Furthermore, the purpose of an ethnographic case study is not to determine the causes of the circumstances, but rather to describe and understand how they evolve (Stake, 1995). Case studies can satisfy one of three purposes in qualitative approaches: describing, understanding, and explaining. At the same time, they can be single or multiple-case

**Figure 3:** Multiple-case design of the study (adapted from Yin, 2009, p. 46)
Figure 3 describes how I use multiple cases to address my research questions. The task is to understand the selected cases and not the population (Stake, 1995; Yin, 2009). In multiple-case research, the development of a case-study protocol that is carefully designed guides the cautious replication of the research objectives. Case studies can include quantitative and qualitative data.

Case studies tend to be selective, focusing on one or two issues that are fundamental to understanding the system being examined. They involve multi-perspective analysis. This means that the researcher considers not just the voice and perspective of the actors, but also the relevant groups of actors and the interactions among them (Stake, 1995). The case study is considered a triangulated research strategy, wherein multiple sources corroborate each other and the findings. Six sources of evidence are often used in case studies: documents, archival records, interviews, participant observation, direct observation, and physical artifacts (Yin, 2009). As I discuss below, I mostly used video data along with other sources. The multiple-case design refers to a replication of an analytical process across cases. Therefore, the same qualitative analytical processes, described below and based on the theoretical frame, were consistently applied across cases (Yin, 2009).

The data analysis process entails developing meaning about the case by interpreting directly the individual’s stance and situation. It evolves through an aggregation of situations in which one searches for patterns that may inform each other and observes for consistencies and/or inconsistencies in specific circumstances. As Stake (1995) writes, “we want to understand behavior, issues, and context with regard to our particular case” (p.78). The analytical process starts from the study of natural events through ethnographic methods. The goal is to identify recurrent patterns and verify them through a process of reexamination of their validity across episodes which match and describe the phenomenon of study, allowing us to generate new insights and explanations and/or
theory from concrete actions (Biddle & Anderson, 1986; Patton, 2001). In addition, generalization from a case study is an analytical generalization grounded on a theoretical perspective. Therefore, the stronger the theoretical bases in a case, the stronger its findings and analytic generalizations (Yin, 2009). Thus, I used both an analysis of empirical actions as well as an analysis supported by theoretical perspectives about the relevance and necessity of a social dimension for teaching and learning mathematics (Zevenbergen, 2000), problem solving (Schoenfeld, 1985) and collaborating (Brenner, 1998) in bilingual (Adler, 2001) mathematics.

D. Data Collection and Analytical Processes

In this section, I describe the students I selected as cases for this study and the criteria and process for that selection. Then I describe the context of the study, the process of data collection and the analytical processes applied.

1. Participants

I specifically focused on the interactions of four students. I selected these students in order to focus intensely on contrastive cases. This selection represented a theoretical, instrumental sample of potential profiles that could better help me understand issues of positioning and participation. The selection criteria for student cases were based on theoretical stances. These theoretical stances refer to the various constructs I decided to explore. The principal phenomenon was positioning, a process which may be subtle and occur in interactions where the same group of participants may act toward one student differently than toward another. This process is negotiated through discourse (Harré & van Langenhove, 1999). The different positions mediate the next construct in this study, participation, which is a process intricately linked to learning and refers to how individuals socially engage in a task or practice (Lave & Wenger, 1991; Rogoff, 2003). Therefore, I explored the quality of students’ participation, the times when they were “in” or “out”
of the problem-solving activity. And when they were “in,” I explored how these students and the rest of the group interacted through discursive practices. I focused not only on how they used language to interact but also on what language they used (English or Spanish).

Therefore, studying the interactions of only one student might not adequately offer the comparison and highlights I wanted to explore. Therefore, I decide to select two students who I selected based on my informal observations during my work in the afterschool and also through my previous studies of the data. I selected two students that seemed to have high status and two with low status in mathematics with the purpose of contrasting their kind of participation as they interacted. I also selected students that participated in the same group. Furthermore, as my research questions revolve around interactions, studying only one group of students’ interactions might not give me sufficient insights. Therefore, I studied two groups of students, each with a pair of high and low status students in them. This situation offers contrasts between groups and between students.

Next, I describe the selection of student cases in depth.

I decided to explore the interactions of four students based on initial observations of all 31 students. I was a participant-observer for over three years at Los Rayos, a situation that informed my selection of cases. My selection represents contrasting examples of student cases based on the following variables: social status within the afterschool, gender, language choice, and disposition towards mathematics. These processes are founded on the idea that the sampling of cases from the chosen population requires a theoretical sampling, or that the cases be chosen for theoretical, and not statistical, reasons; with the purpose of replicating previous cases and studies; to extend emergent theory; or to fill theoretical categories or examples of polar types (i.e., in which the process of interest is “transparently observable”) (Eisenhardt, 1989; Glaser & Strauss, 1967). Thus,
I developed the selection of cases based on the potential manifestation of the theoretical constructs of this study, namely positioning and participation through a process described below.

I developed my case selection first based on Berger, Cohen, & Zelditch (1972) and Cohen’s (1984, 1994, 2000) *Expectations Theory*, which describes that students’ status transfers across settings and activities and that people in the context seem to react accordingly. For this study, I considered pertinent the selection of students that differed in their mathematics performance in the regular classroom (as reported through school grades), attitudes towards mathematics, and language preference or fluency. My selection was first informed by my direct observations. During the development of the program and I interacted with students, I noticed that students commented on each other’s mathematical skills and language fluency and preference. At times, they also compared their activity in the afterschool to that in their classrooms. Therefore, before selecting the four cases I informally selected 8 possible cases that provided contrasting levels of participation.

In order to narrow down my selection, I started considering their attendance to the program. I selected those who attended the program during its entire duration. Next, since one of my interests was on the role of language in positioning, I explored students’ self reports on language fluency and selected those who had contrasting language preferences and/or abilities. These self reports seemed corroborated as I also consulted their school reports. Students who self-identified as balanced bilinguals, they also had the lowest scores in English as a second language. The other two students self-identified as English dominant. Also in their school reports, I checked their grades mathematics and in fact my observations in the program matched students’ school profiles. Those who seemed to have a low status also had lower school grades, and those students with apparent high status also had higher school grades. Table I displays the four student cases selected (two boys and two girls) including their names (pseudonyms), language dominance, school grades, and mathematics attitude.
As part of Los Rayos, a non-standardized mathematics attitude survey was developed specifically for the program and administered at the end of the students’ 5th grade in spring 2008. The purpose was to gather additional information from the students about their experiences in mathematics. This extra information also supported my selection, as there seems to be a correlation between students’ grades and their attitude towards mathematics. Fabiola and Raúl have the highest scores in their reported enjoyment of, confidence and motivation in, and understanding of the usefulness of mathematics. Betty and Orlando have lower scores, that is, less productive attitudes towards mathematics.

Finally, through my fieldnotes from interviews with teachers from the school, I recorded their perceptions about students in the afterschool. Teachers’ informal reports on variables such as a) attachment to students b) their academic performance, c) their behavior, and d) social issues about students, and e) possible indifference about students’ performance (Cook, 2004) also corroborated my selection of cases. In general, teachers reported concerns about Betty and Orlando’s behavior and academic performance. And they reported greater attachment to and confidence in Fabiola and Raúl’s academic performances.

### TABLE II
PROFILES OF STUDENT CASES

<table>
<thead>
<tr>
<th>Student Cases</th>
<th>Self-Reported Language Dominance</th>
<th>Average Mathematics School Grades</th>
<th>Survey on Mathematics Attitude</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Confidence Usefulness Enjoyment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Fabiola</td>
<td>English</td>
<td>A</td>
<td>16</td>
<td>23</td>
</tr>
<tr>
<td>2. Betty</td>
<td>Both</td>
<td>C</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>3. Raúl</td>
<td>English</td>
<td>A</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>4. Orlando</td>
<td>Both</td>
<td>C</td>
<td>16</td>
<td>17</td>
</tr>
</tbody>
</table>
As a result, I believe that my selection of cases represents a theory-based sampling (Patton, 2001) and an instrumental case study (Stake, 1995) which comprises two pairs of contrasting cases. My cases include a pair of students who appeared to have low status in the afterschool, had a history of poorer performance in mathematics, and had a preference for Spanish. As a contrast, another pair appeared to have higher status in the afterschool, a history of better performance, and a preference for English. At the same time, my pairs are found in two different groups. This allowed me to “test” my data analysis across the contrasting pairs. The study of these student cases’ interactions represented the contrasting social and academic experiences of some students in the ecology of doing mathematics problem solving. The goal was not to study the cases themselves but rather to understand the positioning process of contrasting cases during problem solving.

Students interacted with facilitators, peers, and, at times, mothers who volunteered in the program. The students selected for this study participated in the same group most of the time, even though they were free to participate in any group they chose. These groups were usually formed by members of the same gender, but they varied somewhat as students moved temporarily to different groups. The “home” groups seemed to have stabilized after the first semester.

2. **Site of Data Collection**

This study used data gathered for a larger study of an afterschool program (Khisty, 2004). The program, run for just over three years, “Los Rayos de CEMELA” [Thunder Bolts of CEMELA] (Khisty, 2004) was designed as an adaptation of the *Fifth Dimension* (Cole et al., 2006) and other projects such as *La Clase Mágica* (Vásquez, 2003) and *Las Redes* (Gutiérrez, Baquedano-López, & Alvarez, 2001). *Los Rayos* was intended to investigate the language, knowledge, and social practices of Latina/o students while they engaged in non-remedial mathematics in a bilingual and playful atmosphere. It was a hybrid space created by promoting both languages; with a playful
approach, but not play; with a school subject, but not school (Khisty & Willey, in press). *Los Rayos* was also designed as an activity system (Engeström, 1999): an open, evolving, and collective structure. My research interest in the positioning and participation phenomena arose within this setting. Data collected within this project comprise the sources explored in this study.

Participants in the afterschool met twice a week for about ninety minutes each session for about 8 weeks during each school semester. Students volunteered to be part of the afterschool and were encouraged to be as self-directed as possible within certain constraints. They were encouraged to work with other students of their choice and to engage with a facilitator. The facilitator was either an undergraduate Latina/o pre-service teacher or a research assistant. The facilitators comprised an average of about 10 adults (5-6 research fellows and 5-6 pre-service teachers) that varied across semesters, especially within the group of UGs. Facilitators served to guide students, answer their questions, and engage them in extended talk about mathematics.

The participating students, mostly US-born and from a Mexican background, were all bilingual (Spanish and English). 31 students formed the whole body of participants over three and a half years (2006-2009) with an average of 17 students per semester. The program evolved with almost the same group of participants (12 students, mostly girls); it started with their 3rd and ended with their 6th grade. Students came from two different sections of the same grade, and they self-selected to be part of *Los Rayos*. From this pool of students I selected four students as cases for this study (I explain details about this selection above). The program took place in a dual-language public school in a large urban area. The school itself is located in a low-income neighborhood where the student population is nearly 100% Latina/o. The office of Research, Evaluation, and Accountability of the city, as of 2008, reported a population of 98% Latino Students, 91% Low
Income Students, 9% Special Education Students, and 62% Limited English Learners or English Language Learners in this school.

To further understand the context in which the student interactions occurred, it would be relevant to have some background on the kind of mathematical tasks students engaged in. *Los Rayos* promoted a series of different tasks and project-based mathematical activities, which often promoted mathematization processes. Mathematization is a process of familiarization with or the application of mathematics to everyday contexts; it is also a reinvention process in which “students formalize their informal understandings and intuitions” (Gravemeijer, Cobb, Bowers, & Whitenack, 2000, p. 237). The curriculum targeted mathematical concepts such as probability, proportional reasoning, geometry, and pre-algebraic thinking. All tasks were either problem-solving or -posing oriented. Figure 4 portrays *Los Rayos’* timeline of the different units developed during 7 semesters.

**Figure 4**: *Los Rayos’* timeline and curriculum units.

3. **Data Sources**

As this program had already been developed, the data that were already collected and existent became my sources for evidence. Thus, evidence was gathered from various types of data, such as videos of small-group interactions, student interviews, selected facilitators’ fieldnotes,
student work and messages to El Maga, and regular teachers’ informal comments gathered in researchers’ fieldnotes.

The main sources of data are videotapes of the interactions of case study students in small groups throughout the three years of the program. The program ran for 6-10 weeks during the fall and again in the spring semester of each school year. Students participated when they were in grades 3-6. Therefore, there is an estimate of a maximum of 540 hours of video footage. Students were interviewed using unstructured and semi-structured protocols at different times during the program, and these interviews are also included in the video data. These interview sessions include the following: debriefing session (15-20 minutes) at the end of school year 2006-2007; five student interviews (30-40 minutes) at the end of the school years 2007-08 and 2008-09, respectively; and finally, three focus-group sessions (20-30 minutes) during spring 2009. Interview sessions mostly explored aspects related to the purposes of the program: students’ experiences in the afterschool activities, their use of and learning mathematics in and out of school, and their language use and preference (Spanish, English, or both). The questions ranged from semi-structured to open-ended formats; when students brought up ideas or issues that they wanted to talk about, the interviewer followed up that lead until finished and then moved onto another item. Facilitators conducted the interviews either in Spanish or English depending on students’ preference.

4. **Data Organization and Reduction**

I organized the data based on my research interests. I focused on mathematics problem solving with a twofold goal: first, because problem solving promotes situations where students can expand what they currently know (Silver, 1987; Rogoff, 2003) and, second, because it is a social distributed process (Pea, 1993) where competence is situated in social relationships (Lave, 1988; Kirshner & Whitson, 1997). Students communicated, shared, and problem solved by
using language, and in this interactive process they mutually affected their reasoning in mathematical concepts and their participation and positions in the group. My interest concerned not only the functions of immediate interactions in how students were positioned but also students’ positions and participation trajectories over time (one of my sub-questions).

One of the greatest challenges that I faced before starting the analytical process was the reduction of the data sources. I began by cataloguing the data and selecting the number of sessions when student cases participated. This step reduced the total hours of video data by nearly 50%. Then I decided to narrow the number of semesters. *Los Rayos*’ timeline included a total of 7 semesters, but I used five. This seemed adequate since it was the time when these students interacted in small groups. During this time students attended grades 3 through 5. Next, as there were between 10 and 12 sessions per group per semester, I selected 5-6 sessions for each group per semester, for a total of 60 sessions. The selection of these sessions was based on having at least one student with “in” participation during problem-solving activity and on the interactions evolving within a group with at least two of the student cases. In this whole set I included only 2-3 sessions per case when they were not interacting in a group with another student case. The total of 90 hours was reduced by observing all videos and selecting only those segments of the sessions in which students were engaged in problem solving. This procedure narrowed down the data to approximately 38 hours. I organized these data in segments, each equivalent to one semester and a half, in chronological order: a) the beginning, b) the middle, and c) the end of the program.

5. **Data Analysis**

The issue of trustworthiness and credibility is a concern in qualitative research. To address this Howe & Eisenhardt (1990) suggest maintaining a cohesive focus throughout the process by following five principles: a) the fit among research questions, data collection, and
analysis techniques, b) effective application of those techniques, c) coherence of background assumptions, d) warrant, and e) external and internal value constraints.

In the analysis of the selected video data, I developed a categorical aggregation process and direct interpretation of bilingual students’ positioning and participation as they interacted in problem solving in small-group work. As Stake (1995) describes, categorical aggregation is a collection of instances of meaningfulness and puzzlement to perhaps arrive at an indicator of comprehensibility where the qualitative researcher focuses on the instance, trying to pull it apart and put it back together again more meaningfully, searching for meaning and patterns within certain conditions or correspondence. The purpose is to understand behavior, issues, and contexts with respect to a particular case (positioning). After this, the researcher then needs to look for other passages, reflecting, triangulating, being skeptical about first impressions and simple meanings, and searching for the most critical evidence for one’s assertions.

I started my analytical process by selecting and observing some sessions from the beginning, middle, and the end of the program. I focused on the moment-to-moment interactions, exploring the evolution of the participation, positions, and use of language of the selected students. Through constant comparison, some patterns emerged regarding the quality of engagement of students. I grouped these patterns into categories and observed not only whether students were in or out of the social interaction around the problem-solving task, but I also paid attention to the area of engagement or what students talked about or were interested in the most as they interacted.

This could be mathematical or social activity. At times, students would focus on one or two of the areas. Another category that emerged was the quality of engagement in those areas. Students in the same group demonstrated different levels of participation, as some had more central or marginal participation while relating with others in the group. The way the participation was
constructed in the group seemed connected to the ways others addressed students and how students themselves responded to those interjections. The language students were using was also another category that I developed (See Appendix A for a specific description of each category). These categories emerged in an open process trying to capture the nature of the construction of positioning and participation. These initial categories were contrasted and compared across analyzed sessions, and, once established, they were applied as working constructs to code the rest of selected sessions (Stake, 1995; Strauss & Corbin, 1990). Through the categorical aggregation and direct interpretation procedures, I noticed that both positioning and participation were dialectically integrated. Thus I developed an integrated analysis using the set of emerging categories that were enriched as the analysis continued. New patterns (didactic situations or forms of interacting around the problem-solving task) emerged in order to better describe social processes. New categories and subcategories were added and utilized to analyze the video data.

The presence of the lines of power became apparent, especially through differentiating processes that participants used to mark, single out, or position one another. The previous process of marking the quality of participation in the videos helped me identify positioning episodes across the selected hours. These were transcribed and analyzed. Here, I was able to correlate the conditions of positioning (storylines, positions, and speech acts) with nonverbal communication such as gesture and gaze (Bloome et al., 2005). With the purpose of distinguishing the quality of positions, I developed greater categories or types of positions by matching and differentiating the previous patterns. Four types of positions were determined (i.e., powerful, powerless, equalized, and null) and connected to positioning episodes. At times more than one position was co-constructed per student even within one episode. This process contributed to establishing the goal of the study and
was applied to and documented in all sessions. Codeswitching became relevant in the process of co-construction of positions. Thus, I further explored patterns of language use during positioning.

In the analysis of the role of language, I explored times when, within the same conversation or interactions around positioning events, the participants codeswitched to another language. I categorized the different functions served in the process. Similarly, there were episodes wherein students argued and challenged each other with the goal of achieving or understanding the task using the two languages. This process led me to explore the frequency of language use by students and how the rest of the participants in the group responded and addressed these students in the completion of the problem-solving task. For this, I coded the frequency of turns per minute in either Spanish or English and the relation to the kind of negotiations developed in the group centered on a specific student case. I paid attention to who spoke to whom and in what language. Again, the purpose of this strategy was to explore the use of language and its functions during interactions.

The analysis of positioning and participation over time was done by tracking students’ accumulated positions. I developed this analysis by both focusing on the total frequencies of the different positions and the type of positions per student case, as well as what these meant in relation to the total number of positioning events analyzed, by using percentages. For this, I applied Yin’s (2009) techniques of time-series analysis. In order to simplify the analysis, I converted the five semesters that I analyzed into a time series with three segments: a) beginning of the program, b) middle of the program, and c) end of the program. Each segment approximately corresponded to one semester and a half of analyzed data distributed in chronological order. Thus, the analysis of students’ positions included the mapping of their position trajectories across the three segments in the *Los Rayos* timeline. Thus the microethnographic analysis of the positioning of each student case (Bloome et al., 2005) was converted into percentages of positions during each of the segments in
the time series, which were represented in cross-case displays and patterns were compared and
corrasted across cases, time series, and plausible and rival explanations (Miles & Huberman, 1994;
Yin, 2009). To understand positioning better, I used Activity Theory (Engeström, 1999) to explore
the possible contradictions that emerged within and between the different Activity components (i.e.,
object, outcome, division of labor, community, tools, and rules) during each of the different types of
positions.

The process of comparing and contrasting was supported by developing an outline of the
findings and chapters (Wolcott, 2009), which was continuously discussed, negotiated, and double
checked. In addition, the patterns or categories utilized in the analysis of this study were also
triangulated against other sources of data such as the pre-service teachers’ fieldnotes and what
students mentioned during the interviews. As Rossman & Rallis (2003) recommend, “write all the
time.” I developed my hunches, thoughts, and impressions through various memos developed
during the analysis mentioned above. These memos allowed me to track my own sense of the
analytical procedures of the data. The process of narrowing down the patterns of interaction around
positioning only became evident through contrasting a set of positioning events, and after a pattern
emerged, it was extended to analyze similar relations in the rest of the episodes. In the findings
chapter I present and discuss three factors mediating the positioning processes.

As a reminder, Eisenhardt (2005, 2006) claims that an important element in research is its
trustworthiness, which is achieved by presenting evidence of various aspects: having been there
(direct participation with the scenes of action) and validity (accurate interpretations). Qualitative
researchers should resist “the conception of science as a body of finished propositions derived from
empirical research or a set of formulations to be applied as technical rationality to the shifting and
induplicable situations in which people live” (Greene, 1997, p. 190). The situation should leave open perspectives for the future and not consider them as finally understood.

Summarizing, I utilized various research strategies described above, such as theoretical propositions (previous theories informing the study), case descriptions (using multiple and consistent forms to describe and analyze cases that were theoretically sampled), and rival explanations (inclusion of plausible contrasting perspectives on the phenomena, conditions, and explanations) (Yin, 2009). These strategies were continuously informed not only through my analysis of the data, but also through my participation and direct observations over three years in the program. In order to enhance the credibility of the results, I triangulated them through multiple sources of data and also investigator triangulation (Stake, 1995). My multiple sources were comprised of my fieldnotes and informal comments about the specific students in my study; fieldnotes from pre-service teachers and other researchers in the afterschool; archival records such as students’ school records; results from a mathematics attitude survey; my data catalogue; videotaped individual and focus-group student interviews; videotaped interactions of participants throughout the program; and student work. For the analysis of fieldnotes and my own memos, I used NVivo and Excel for cataloguing and coding video data.

E. **Summary**

In this chapter, I described the qualitative research methods I used for this study. A qualitative study seemed most appropriate given the research questions and the nature of the studied phenomena in a natural setting. I demonstrated that an ethnographic study of the selected student cases demonstrates that they are a potential source for the exploration of meaningful results in the positioning and participation of bilingual students during mathematics problem solving. Thus, my selection of participants was consistent with my theoretical stance and my methods of data analysis.
The sources of data and the research design support an adequate analytical process. The comparative analysis of the positioning and participation patterns across student cases and of the patterns over time offers a rich opportunity to explore the relevance of the quality of student interactions and language use during mathematics problem solving. This analysis will also identify patterns of collaboration among students, facilitators, and tasks and how each intervenes in the process of positioning and how these relations may at the same time mediate the ways students see themselves as bilingual doers of mathematics. In the following chapter I present and discuss the findings developed through this analytical process.
V. FINDINGS

A. Overview

In this chapter I discuss the findings of my investigation of interactional processes in small groups during mathematical problem solving in response to this study’s primary question:

*How do social interactions in small-group work mediate the positioning and participation of bilingual Latina/o students during mathematics problem-solving tasks in an afterschool program?*

I have organized the discussion of this question according to three sub-questions:

A) How does positioning emerge during social interactions around Problem Solving?
B) What role does language play in positioning?
C) How do positioning patterns change over time?

This study was situated in an afterschool non-remedial mathematics club and explored the social interactions of four students. By social interactions I mean the actions and relations that students and adults developed by taking part in a group together during their work in the afterschool. The analysis focused on the students’ actions and discursive practices that appeared to contribute to the emergence, co-construction, and negotiation of their positions, roles, and actions during problem-solving tasks by discussing, compromising, agreeing, and/or disagreeing with others in the group. These students represented contrasting cases. Through my observations in the afterschool, it was clear that two of them, Fabiola and Raúl (female and male), were considered to have high social status. Facilitators and peers gave these students more attention than other students in the group. Additionally, these students had average to high grades in mathematics. The other pair, Betty and Orlando (female and male), seemed to have low social status, as their concerns and actions did not receive much attention from others. Others sometimes referred to them as someone who misbehaved or did not know enough mathematics. In addition, they both had low to average school grades in mathematics. As I discussed in the previous chapter, I limited my investigation to only five of the total semesters in the program, and I focused on the instances when students
engaged in doing mathematical problem solving and not in other interactions such as play or personal conversation. The small student groups often included adults, who were either research fellows or pre-service teachers. These adults clarified, responded to, and prompted students as needed during the tasks, thus facilitating the mathematical work in the small groups rather than actually instructing them. Students usually selected the facilitators they wanted to work with. At times, a group of students’ mothers also participated. Since my focus is on student interactions, I identified adults in this study simply as either Facilitators or Mothers.

Essentially, in this chapter I describe the discursive emergence and negotiation of student positioning and participation through social interactions in relation to three factors I identify as alignment, attention, and ability. Next I describe how power came to be associated with language (mainly English) as the students negotiated their positions during their interactions in problem solving. Then I describe positioning and participation patterns that appeared persistent over time and parallel to students’ academic histories. Despite these continuous patterns, I discuss the situated, flexible nature of positioning, as it varied through discursive negotiation and development independent of students’ previous academic status. Lastly, I triangulate these trends with students’ reflections on their experiences and self-perception as bilingual doers of mathematics.

Before presenting the results, I recall here relevant definitions in the study. Social interaction designates jointly and dynamically co-constructed, discursive activity geared to meaning making in which persons produce activity and co-construct each other’s positions (Harré & Gillet, 1994; Lave & Wenger, 1991; Rogoff, 1990). These processes are especially relevant in mathematical activity (Gonzalez, et al., 2001; Zevenbergen, 2001), as they support and promote student participation and mathematical understanding. Participation refers to the way individuals socially engage in a task.
Positioning, dialectically related to participation, refers to how individuals are perceived by others and how they present themselves in relation to one another and the activity.

B. How Does Positioning Develop in Social Interactions around Problem Solving?

In this section, I present examples of students’ positioning and participation and describe how these evolved during problem solving. I selected examples or episodes that would both represent the positions and participation of the four focus students and that would portray general and contrasting patterns of positioning processes. The study’s students participated in two groups of their own choosing, one composed of males and another of females. The analysis of how social interactions mediated positioning yielded patterns that suggested that positioning and participation were linked to the interplay of the following factors: alignment, attention, and ability. Despite the continuous interplay of these factors, I present each factor separately and in detail and elaborate on each with specific examples. In general I use alignment to refer to how participants’ actions come together by coordinating with and elaborating on what they or other participants say or do. Attention refers to the process of selectively noticing certain aspects during an episode or situation. It determines what matters to participants at a given moment. Finally, ability refers to participants’ socially supported and constructed competence.

1. Alignment

In this section I describe alignment and present examples of how this factor relates to the process of positioning. I use alignment to mean how participants’ actions come together and match or mismatch as they share the same social space and/or activity. Alignment manifests itself through what participants do and/or say to each other, responding to each other and converging with the same goal, understanding, and/or interest, thus forming a collective activity of shared actions or goals. Alignment is constructed when one’s actions in the activity either parallels or intersects with
the actions of others, a process evidenced by how one aligns or agrees with and/or elaborates on what others say or do, and similarly, by how others align with and/or elaborate on what one says or does. Moreover, the alignment of actions can include the emergence of possible disagreements that participants focus on (attention), but which they still can negotiate. In regards to joint activity, Matusov (1996) argues that agreement is only one phase in the process of working together; disagreement also evidences social processes of coordinating each other’s actions around a goal. Therefore, although the process may result in divergent pathways, the process of negotiating differences indicates the convergence of the participants’ actions and goals—that is, their alignment—regarding a topic at certain points. Then, this negotiation or coordination of actions could result in the continuation or the interruption of alignment. I discuss the emergence of positioning and alignment by exploring these two dimensions: a) alignment among participants and b) alignment with a task.

a. **Alignment among Participants**

In this section, I describe how alignment among participants was constructed during students’ interactions while doing mathematics. I present two examples in which, despite the different positions and participation of members in the group, their alignment in what they said, did, and understood (or not) mediated an integrated meaning process that equalized their positions.

The first example stems from a situation in which a group of 4th grade girls, Betty, Fabiola, and Elsita, as well as a female facilitator, decided to play some games. They selected the Hanoi Tower puzzle, which had written instructions in Spanish and English. From the beginning, the students’ different positions became evident. Fabiola, the student with high social status and high

---

2 The Hanoi Tower puzzle has a board with three pegs, and on one of them is a stack of 7 disks of different diameters. The goal is to move the entire stack onto another peg with the fewest moves by: a) moving only one disk at a time, b) taking only a top disk from one peg to another, and c) always placing a smaller disk on a larger one.
school grades, was called the “expert” by the facilitator, since she already knew how to solve the puzzle. Fabiola was asked to explain how to solve the puzzle, but after her explanation, everyone (including the facilitator) still seemed confused. The players decided to take turns and get feedback from others in order to learn how to solve the puzzle. Fabiola still noticed confusion and decided to explain again, but this time using fewer disks. The simplified moves made it easier to understand. The group played together, assisted by Fabiola, and successfully completed the game. Both Betty and the facilitator attempted to solve the puzzle independently, but neither was successful. At this point the group was at a powerless position in relation to the task since they still did not understand the basic process to tackle the puzzle. With powerless positions, I mean a position in which one lacks the power or the opportunity to make decisions, understand, and carry out the task in an active role. Fabiola noticed these difficulties and coordinated her actions with the group’s concern by playing along and solving the puzzle with them. The process of coordinating each other’s actions in the group promoted new positions for Betty and the facilitator, in that they all engaged in the same problem-solving activity at a similar level and shared the process of solving the puzzle. Their positions in the task were more equalized as the power of knowing was distributed among the participants in collective action.

Alignment among participating students emerged through a process of collaboration around their genuine interest in wanting to understand and solve the interesting and challenging puzzle. Fabiola, based on her prior knowledge, played the role of an expert by providing explanations and adjusting the task’s level of difficulty (i.e., reducing the number of disks) based on what Betty and the group were saying about or doing with the puzzle. Fabiola knew that having fewer disks would make her explanation clearer and easier for her audience to comprehend. Fabiola closely followed Betty’s moves, trying to understand them and suggesting alternative moves accordingly that would
more directly lead to the solution. Betty seemed to need assistance and sought Fabiola’s advice; arranging stack of disks, Betty says: “OK! Let me practice. Let me see how I go.” Fabiola, placing the last disk on the stack, responds: “Let me tell you as you go along.” Nevertheless, Betty was actively making sense of the process, which afforded her a better understanding of how to solve the puzzle. This example portrays participants’ actions as they aligned: Fabiola, the “expert” (with high social status and grades), and Betty, the non-expert (with low social status and grades), aligned in the achievement of a shared goal through collective processes. By talking with Fabiola and trying various strategies to solve the puzzle, Betty made sense of the task. Fabiola, the expert, observed and listened to others and adjusted her explanations and the task by working along with the others. The others, including Betty, observed and listened to Fabiola while they actively engaged in and came to understand the task. Thus, alignment among participants’ interactions was supported by a process of willingly coordinating the different levels of action and understanding the task toward a collective goal.

The second example describes how the group’s interpersonal interactions, or collective alignment, in trying to understand how to play a game involving frequency patterns actually helped everyone’s reasoning about probability. In this situation, the group was comprised of seven people: Raúl, María, and Orlando (3rd graders), each student’s mother, and one facilitator, agreed on playing the game. They played a total of five rounds of the Counters Game that required participants to recognize a pattern of frequency in the numbers that came from rolling dice in order to win the game. During the game most of the discussion was in Spanish. The interaction was animated and almost after each roll one could hear “Yay! Yay!” or “Boo! Boo!” depending on how the roll of the dice affected their chances of winning. Participants’ actions and comments were coordinated with others by collectively elaborating on what others did, the strategies they used, and the relevance of
the number seven in the game. As the group played, they helped each other understand how the game worked by observing and commenting on each other’s moves. Raúl won the first rounds, and this drew attention to his counter distribution. Others’ remarks recognized Raúl as a winner and pointed out his successful strategies for winning the game. Transitioning into a new round and with the purpose of making the previous remarks clearer to all, the facilitator probed participants about the recurrent patterns: “Why do you think some [numbers] come out more than others?” Orlando replied “I don’t know!” This statement may depict either how the processes embedded in this game (i.e., noticing number patterns and frequencies) were still not obvious to Orlando or simply that he did not know how to articulate his ideas. In either case, with that remark Orlando positioned himself at the margin of understanding and carrying out the task. His stance implied a distance or disadvantage in relation to the task. Orlando then had positioned himself at a marginal or powerless position, a situation which was not reinforced by the group. Instead, Raúl replied:

**Raúl:** Con el dos no sale mucho porque, casi no hay muchas posibilidades para hacer uno y uno, con el dos. Y tampoco con el doce. *The two does not come out often because there aren’t many possibilities to make one and one, for the two. And not for the twelve either.*

As Raúl explained his mathematical realization, the facilitator further prompted him, asking about the high frequency of the number seven, and Raúl explained: “*Because there are more possibilities for it to come out, like five-two, four-three, and so forth.*” Though the number seven had already been collectively constructed as an important one in the game, only Raúl articulated the patterns and reasons that made seven the most probable sum in the combination of rolling two dice with the numbers 1-6. Raúl established himself as having a central role in the game since, through playing the game with others, he understood and was able to articulate the patterns that led to winning the game. He played and ‘discovered’ the mathematics by noticing and observing the number patterns that the group was using and commenting on. The way Raúl positioned himself contrasted with
Orlando since they denoted different levels of relating with and understanding the task. Up to this point, these students’ actions had positioned them at different levels. Nonetheless, the rest of the participants or the social interactions in the group did not elaborate on this difference. Rather, the group continued coordinating their actions and comments with each other and the task and also with what Raúl had mentioned (i.e., how attributes of the number seven were relevant in this probability task). Thus, the actions of the participants aligned in interactions that socialized and collaboratively elaborated on ideas that helped everyone understand the game and the winning patterns. The relevance of seven became a collective tool and strategy that all used during the game. Therefore, this quality of alignment among participants placed emphasis on the collective meaning process more than on whose claim it was, which ameliorated different emerging positions. This process evolved to the point at which the whole group decided to exclude the number ‘seven’ from the game. A mother claimed: “If we all keep choosing the seven, we all are going to be tied.” Although the differences among participants could have been enhanced or made more obvious, it was not the case. The group collectively managed and shared their understandings which afforded similar levels of noticing and using probability patterns; thus the process equalized their mathematical positions as knowledge was distributed in the group. The emphasis was on the collective goal, and individual contributions, such as Raúl’s, became part of the group’s repertoire that all accessed as needed.

Interactions in both previous examples relate to Vygotsky’s concept of the zone of proximal development, in that interpersonal processes mediated higher levels of solving a puzzle and understanding concepts of probability, respectively. By interacting with “experts” like Fabiola or realizing mathematical patterns as they played as in Raúl’s group, the rest of participants eventually appropriated the external processes into intrapersonal ones to lead their own actions (Vygotsky, 1978). During the interaction some students were still experts and novices, but their acting together
and their collective alignment around meaning making was the most important element in problem solving. The presence of a collective goal afforded participants with opportunities to engage in problem solving in a collective and active endeavor that coordinated them despite their different levels of understanding and solving the task. Thus, these processes equalized the different positions and mediated interactions of collective activity. Even when Fabiola directed and negated Betty’s moves, her actions were a coordinated response to Betty’s; her statements and directions provided the necessary feedback for Betty to complete the task. Thus, alignment with the task and each other is maintained even when participants disagree on certain elements, as long as their overarching goal remains the same. What participants observe, listen to, and do is directed at connecting with others and building from there. Even when Orlando could not respond to the challenging prompt and Raúl did, the emphasis was not on who did what, but on what it meant. Thus, there were game winners and losers, that is, different positions, but that was not the object of focus. The alignment was a collective one with a collective goal and a collective product. In fact, when the focus of attention started shifting to who was winning in the girls’ group, this process increased the differences, and the respective powerful and powerless positions became obvious.

b. **Alignment with Task**

I now describe two examples regarding dis/alignment with the task. This section describes how some students may have “the choice” to change the course of actions in a group successfully while others are marginalized from this process. It also describes the different levels of role flexibility for participants with respect to their mathematical positions that evolve or are co-constructed during problem solving. In the first example, I present how Raúl shifted roles during a task and how others supported this process. The group was comprised of Raúl, Orlando, Mario (4th graders), and a male facilitator who decided to play a board game called the Grocery
Cart. The game, similar to Monopoly, promoted concepts of estimation, decision making, and applied mathematics by making the best purchases. Soon after the game started, Raúl (a student with high social status and grades) seemed bored with the game as well as with his role as a player; he also had corrected a peer for not having done a proper job as a bank manager. Raúl suggested playing a new game, but as the group refused, Raúl negotiated a shift in roles so that he could be the bank manager. The group accepted this shift, and the game continued. Although new roles had emerged in the group, the kind of interactions in the group remained very similar, in that Raúl still led the game and supervised and approved what others were doing. Raúl’s fluidity of action and accuracy of decisions showed that he already knew the game’s rules and constraints, which gave him an advantage over the others. These circumstances gave Raúl a certain level of authority in comparison to the rest of players that helped him lead the group. The group, but especially the facilitator, recognized this and relied on Raúl to play the game. These circumstances empowered Raúl’s actions and eased his transition into bank manager. Indeed, his peer readily relinquished this role. The new position in the group as the bank manager gave Raúl the official power to manage and control the game and his peers’ actions. In his new role, he efficiently fulfilled all his duties, such as giving exact change, advising about what to buy, and keeping track of whose turn it was to play. Raúl developed a central role and powerful position, supported by the facilitator and peers, since he not only knew what to do but felt free to make changes that best fit his interests.

With this centralized power, a disagreement emerged between Orlando (a student with low social status and grades) and Raúl since their addition of some purchases did not match. Although the facilitator reviewed both answers and both were correct, Raúl walked around the table and pointed to the catalogue and to Orlando’s work and said: “Because look it! You, you put the wrong price! You put the wrong price! I knew it, I knew it!” For some reason, Raúl assumed that he
himself could not be wrong, but only Orlando. In a sense, Raúl saw himself through these circumstances (and also perhaps through previous work together) as having greater mathematical accuracy than Orlando and used it to challenge Orlando even without knowing whether there was an error. In the end, Orlando’s mistake was not due to a mathematical error but rather to his unfamiliarity with using the catalog, yet there were expectations of errors about Orlando’s actions and not Raúl’s. These different expectations seemed to help Raúl reposition himself or disalign from the role that he did not want and instead take the role that he preferred. These expectations contributed to assuming Orlando’s limitations in mathematics and positioned him as someone who made more errors and knew less mathematics. As a result, all participants in this interaction coordinated their actions around the task, but not exactly with each other. The facilitator’s actions simply reinforced the completion of the task. This gave Raúl the opportunity to shift roles and manage the game since the task supported what he knew, but others less familiar with the task had more fixed roles and were linked to deficit-laden assumptions. Both Orlando and the peer who was the initial banker gave up their roles without much resistance, for they seemed to have recognized and accepted the task-related impositions as well as Raúl’s input. Raúl’s situation, however, made him feel empowered, with greater levels of agency to negotiate changes in the task and promote situations that would benefit him more. Therefore the quality of the task and the interactions that evolved around it privileged the student who knew and empowered him (Raúl) to make decisions; the knowledge was centered on one person and not distributed equitably.

In the second example, the target task significantly affected how participants coordinated their actions or not. Contrary to the first example, the disalignment of a student (Orlando) marked him as different and led to his marginalization from the task and the group. The group was comprised of Raúl, Orlando, and Mario (4th graders), two mothers, and three male facilitators (one
pre-service teacher and two graduate students). They visited a local florist shop so they could see the practices there and describe them in mathematical terms. This process is called mathematizing, and it is not an easy process. It is especially challenging to students who are used to doing mostly computational tasks. Mathematizing goes beyond developing a problem. It requires a process of uncovering the mathematics embedded in a practice. The situation becomes even more challenging when the practice is not familiar to the student. That was the case in this mathematization project, as students explored the practices of people working in local businesses. The goal for the students was to examine the mathematics used in various jobs and then demonstrate this in a digital story.

Students interviewed the florist and thought about the mathematics in making flower arrangements. In this process the facilitator (a pre-service teacher) asked students to come up with possible questions based on their first visit. Later they would have a second visit to interview the florist and explore possible mathematics more deeply. The facilitator did not support or help students much.

When he asked students to start working on the questions, Orlando reacted as many students would: “I don’t know math.” Orlando did not seem to know how to articulate the questions, and in response the facilitator provided vague instructions and simply asked more questions:

**Facilitator 1**: I want you to take this pen and write questions. Something you want to know more about. You have some questions here [notebook] and add more. Raúl, I want you to write questions. Questions about something.

Although the facilitator used Spanish to clarify some questions, his support was similar to what he had already said in English. Neither helped the students to start thinking about this difficult task. Only one of three students, Raúl, responded by saying: “I got a math question!” This situation established him as different, as someone who knows mathematics. Orlando and the other peer did not know what to do, and this placed them at a disadvantage. The nature of the task advantaged one student and limited others. Orlando decided to look at the pictures they took instead. This
differentiation did not reside in the students themselves but in the complexity of the task and how it was addressed.

A more expert facilitator joined the group and asked a different type of questions. Orlando engaged more because the questions made more sense to him, for example: “What part did you like about yesterday?” At the beginning Orlando could not understand because he was not as engaged, but when he understood he wanted to jump in and take part in the conversation. Yet he had already been identified as someone who was not interested, and the facilitators gave more attention to Raúl and the other peer, Mario, than to Orlando. The interaction continued, centering on thinking about the mathematics that the florist used, but each facilitator talked to the student (Raúl or Mario) sitting right next them, and Orlando was ignored for over 6 minutes. Later on, Facilitator 1 stated: “I have ignored you because you don’t do your work, when you do it, then I will pay attention to you.”

Later on, as the mathematization project developed, Orlando was able, with the help of the facilitators, to finish his digital story, as did the other students. Nonetheless, this story depicted mostly what he had observed and one or two questions that he heard from others. Orlando was never able to truly engage in the mathematization process, as the facilitators helped him to fulfill the task but not to think about the situations as he had seen them (Khisty & Viego, 1999). This situation contrasts with Raúl’s case, as he created a flower arrangement with the assistance of Facilitator 2. Raúl modeled the arrangement through charts and diagrams, mathematized the process, and created a prediction table for the number of flowers for vases with different diameters. Orlando insistently tried to be part of this process and receive the same type of attention, but he was pushed away. It seemed as if the facilitators were more likely to coordinate or align their actions with the student who showed more understanding and success in the task as opposed to those who did not.
The main point in this example is that the task was not a simple one. It was not straightforward even to the facilitators. And as the task did not make sense to Orlando, his only reaction was to say “I do not know math.” He could not exactly explain what he needed in order to receive the help necessary to do and understand the task. Later on, when Raúl was developing a flower arrangement, Orlando was really interested in the process because it seemed to make more sense to him. Nevertheless, it seems as if the initial part of the project contributed to identify Raúl as someone who knew and was interested in the task. Conversely, this process identified Orlando as someone who did not want to know. This situation started the tendency of assigning different positions to students, which in turn promoted the differentiated quality of attention given to students. This process was based on certain assumptions that were triggered and informed by initial interactions that carried over into later interactions and, in turn, affected how students were positioned in relation to a task and in comparison to other students. Then Orlando had a socially and mathematically powerless position and a mathematically marginal participation and Raúl a powerful position and participation, and both were socially supported by all participants.

Alignment is a multidirectional process in which all participants work to make their own actions converge with those of others during a task. This previous example shows that Orlando was expected to align much more with others, than others were expected to align with him. Although Orlando was ignored by the facilitators, he was often the focus of attention in the interactions of the group. Yet there was little or no alignment to his concerns, needs, or interests. The fact that he was “out” of the conversation did not mean that he was “out” of the social interactions. This process sent direct messages such as: “You do not know the math,” “You need help,” “You are misbehaving,” and “You are not like your peers.” He was constructed as someone different who deserved to be
ignored. Thus, it seems crucial to reconsider the strategy of “ignoring” “misbehavior.” The process of ignoring is not entirely “not noticing,” but a process of marking or stigmatizing someone.

c. Summary of Alignment:

Alignment among participants seemed to develop in relation to how people in a group observed, listened to, and tried to coordinate their actions with others. This situation promoted the distribution or sharing of understanding, power, and action among the group participants, thus equalizing their positions. This also promoted the development of collective goals and work. Such situations were harmed by individualistic interests or comparisons among students. In addition, the process of following a task without accounting for the participants’ needs promoted powerless positions for those students who were asked to perform without understanding. These circumstances often promoted these students’ lack of actions and coordination with others since they were unsure about what to do. These actions of withdrawing were interpreted, especially by facilitators, as lack of interest or knowledge, which led to marginalizing students during interactions and privileging those whose actions indicated some level of knowledge. Students who demonstrated knowing more about a task had more support from the facilitators and the group or were legitimized to choose different roles during the task. The rest of the students had less social support or narrower or fewer chances to introduce shifts, and thus their scope of action, freedom, and roles were limited in the task. This process informs students on what they can do; in turn, it also empowers only some of them to be proactive and make changes to conveniently reposition themselves. Previous studies have found that repositioning came about as a result of a difficulty that some students encountered as they tried to complete a task (Roebuck, 2000). This process evolved in the context of students learning a second language, and it was portrayed as a reasonable way for learners to identify a task beyond their language abilities. Thus, students reframed their positions to prevent being the object
of scrutiny. It parallels Orlando’s experience during the mathematization project as he distanced his actions from the process as he said: “I don’t know math.” Here, then, there were fewer chances to “push” him do mathematics since he “do[es]n’t know.” Contrarily, in Raúl’s case, he placed himself at the center of the interaction and took a chance of being the object of scrutiny. He knew what to do and felt empowered to do so. According to Roebuck, as agents, individuals seem to shape their activity according to their goals, motives, and sociocultural histories. I, however, see this process not entirely founded on a personal choice, but also previous socialized experiences that informed students about what they perceive they can or cannot do well and how to act accordingly. Repositioning or a change of alignment during interactions seems linked also to a shift in the social focus of attention, which pertains to the following section.

2. **Attention or Noticing**

In this section, I describe how the attention of the participants regarding the task or one another affects the positioning process. Attention refers to the act of noting or observing, the process of selectively focusing on one or multiple aspects or perspectives in the environment. During interactions, attention may relate to actions that lead to a focus of attention shared among participants or not. As the focus of attention emerges, other aspects are often ignored. During interactions the focus of attention of a group or a person determines what matters at that moment. Thus, people sharing the same physical space and in coordinated social interactions may still focus on different elements about the task or each other. The ideal scenario would be that alignment and attention go hand in hand, actively mediating and informing the social interactions of the participants. Therefore, attention creates bridges or disconnections that can be elaborated on. As in the previous subsection, I will also discuss positioning and attention in relation to how it develops a) among the participants and b) in relation to the task at hand.
a. **Attention among Participants**

In this section, I present two examples, one from each group (i.e., the girls and the boys). The first example shows how the quality of attention distributed in a group helped students work and solve the task together; it was especially the case for a student who was initially confused. The second example shows a shift in positioning that paralleled a shift in facilitators’ attention to a marginalized participant, thus helping her take a central role in the problem-solving activity and in the group.

The first example comes from a group comprised of Orlando, Raúl, Mario, Alberto (4th graders) and two facilitators. They decided to work on a game involving secret codes. Here, Orlando (a student with low social status and grades) was successfully supported by the attention he received from facilitators during the problem-solving task. Orlando was listened to, encouraged, guided with prompts, and left to work independently. The challenge was to figure out the easiest way to memorize a 36-letter string containing only two letters (S and L). The string pattern read as follows: SLLSLSSLLSSLSSLLSSLLSSLLSLS. The quality of attention given to Orlando helped him refine his observation of the patterns and self-check his answers, but still he worked independently and was actively engaged in understanding how to identify patterns. The group started with each student working separately after reading the instructions. Thus, there was no coordination of actions or collective alignment among the participants, but only with the task. Orlando seemed confused, as he was continuously erasing. But this time, posterior social interactions allowed students to share their ideas and collectively solve and understand the task. Orlando’s attention first emerged when he overheard another peer reading: “SLLSLSS.” This seemed to help him to carefully examine the string, and eventually he found a pattern. As Orlando worked, he at times was assisted by Facilitator 2, who was sitting next to him. The facilitator
prompted and praised Orlando as they worked. Once Orlando had identified the patterns, Facilitator 2 asked: “I wonder if there are more identical patterns in here?” This prompt helped Orlando hone the way he was identifying the patterns and split them in half, which resulted in a six-letter set that repeated six times. Excited, he presented his findings and rechecked them on his own when prompted. Orlando’s work again was acknowledged and praised by Facilitator 1, who was coordinating the group and who introduced Orlando’s work to the group as follows:

**Facilitator 1:** (points to sheet) Look at what Orlando found: one, two, three, four, five, six patterns! And the pattern goes: SLLSLS, SLLSLS, SLLSLS, SLLSLS, and on, and on. Which would be easier to remember, this (Orlando’s) or this one (Raúl’s)?

Calling the group’s attention to Orlando’s work not only acknowledged and elaborated on his ideas, but it also helped the group to notice an efficient way to memorize the string. Raúl concluded that Orlando’s pattern was easier to remember than his, since his was longer. Orlando celebrated. Both facilitators paid attention to multiple issues concerning the task and the students. This supported Orlando’s needs, interests, and successful performance in the task. It also helped him to clear up his confusion and share his thinking. This process granted him a powerful mathematical position as he understood and took control of his mathematical actions. He had both social and mathematical support and attention from others, a situation which made him successful.

This example contrasts with previous situations in which Orlando had powerless positions in mathematics, as he was confused by the task, marginalized from interactions, and had low quality of attention from facilitators and peers. At those times Orlando did not appropriate the goal, but a series of unproductive ideas about self. Raúl successfully navigated the demands of the tasks from the start, which granted him more attention and alignment from the facilitators. In the example, the quality of attention given to Orlando by acknowledging both his accuracy and supporting his limitations not only mediated his better understanding of the task, but also his successful
performance, and it made him interested in the task as well. This process shows that Orlando was not only doing the task, but also appropriating the goal of the task and, at the same time, developing productive relations and dispositions towards the mathematical task. The interactions similarly supported the other peers in the group as they coordinated their ideas and actions to understand and solve the task. A feature relevant to the task in this example is that it had a set of defined steps or a structure to complete the task. The solution, however, was semi-open since there were a few possible correct answers. In this type of task, Facilitator 1 seemed more eager to adapt assistance to students with divergent actions, such as Orlando. Contrarily, during the mathematization task (i.e., an open-ended and unstructured task), the same facilitator had a difficult time assisting and providing similar levels of attention to students who struggled (Orlando). Instead he further supported only those who seemed to understand better, like Raúl.

The second example is from a group comprised of Betty, Elsita, Fabiola (5th graders), and two facilitators who worked on a proportional reasoning task that asked students to make and compare different ratios in two mixtures of orange juice. The goal of this open-ended task was to have students explore ratios and proportions by tasting and selecting their preferred mixture and creating a new one. In these interactions the facilitator’s attention to Betty reshaped the quality of Betty’s interactions with the group. Earlier she was excluded from the mathematical conversation, as another peer dominated the process. When the facilitator acknowledged Betty’s mathematical contribution, the reflections in the group became interesting to all, and participants expressed and negotiated their ideas about creating the mixture. During the actual task, students used utensils (cups, spoons, etc.) in order to define a unit of measurement (not indicated in the instructions) and create the mixture, as well as to decide how much of the mixture they would share amongst themselves. In contrast with the example concerning patterns, this group of girls started working
collectively right away. They focused on the task and commented on each other’s actions.

Nevertheless, from the very start, Fabiola understood the written task and knew how to use the different utensils. Betty was confused with the tools and made that fact explicit to the group. Her peers replied: “Think, Betty! Think!” There was no clear attempt to understand Betty’s concern or support her. In other words, there was no attempt to align with her. But Betty attentively observed Fabiola leading and making decisions about the creation of the first mixture (5 teaspoons of water to 1 teaspoon of orange concentrate). Although the group was collaborating, the mathematical focus of attention was on Fabiola, who also dominated the interactions. Betty, who participated peripherally yet actively observed, started understanding the task, and, though she interjected several times, there were no direct responses to her mathematical ideas. Thus, all the attention centered on Fabiola, and Betty was marginalized from the process of making the juice. She wanted to take part in it, but she could not do so. This process placed Betty in what I consider a powerless mathematical position because her actions and intentions to take part in mathematical decisions were ignored and she could not effect any change. She simply needed to accept the situation as it was. However, she had an active social position since others noticed and responded to her jokes. Nevertheless, while developing the second mixture, the situation shifted and Betty’s mathematical ideas were acknowledged. It evolved as a facilitator noticed what Betty pointed out about making the second mixture. In this second mixture, Fabiola wanted to use the same unit (teaspoons) just as in the first mixture, but Betty—who was still being ignored—noticed that this unit of measurement would not work and therefore challenged Fabiola’s idea:

Betty: (looking at what Fabiola does, she argues) Pero, we have to put more, porque es nada más uno y no va a completar para cinco. / But, we have to put more because it’s only one [unit] and it’s not going to be enough for five [people] (gestures five with fingers).
Facilitator 1: (interrupts and shows sheet to Fabiola, points to Betty) Ooh! ¿Escuchaste lo que dijo Betty? / Oh! Did you hear what Betty said?

Fabiola: (looks at Facilitator 2) Uh? (looks at Facilitator 1, then at Betty) ¿Qué? / Uh? What?

Betty: Que no podemos hacer, no podemos llevar lo mismo porque hay cinco personas y él nomás hizo una. / That we can’t do, we can’t take the same because there are five people and he only used one [unit] (gestures numbers with fingers).

Fabiola: (listens to Betty, then instructions) Oh, so need; so, oh! That’s right! (touches head).

Betty: Porque mira, él hizo una y esto es poquito y no va alcanzar para cinco. / Because look! (points to sheet then to orange concentrate) he made one and this is only a little bit (points to water) and it is not going to be enough for five [people].

Fabiola did not acknowledge Betty until Facilitator 1 publically validated Betty’s interjection. In fact, Betty needed to repeat her claim several times. The facilitator’s move created a new focus of attention that supported Betty’s mathematical positioning and participation. Betty’s claims, now within an empowered position, became important and logically and mathematically legitimate enough to be considered by the group. The group discussed and accepted Betty’s claim, which changed how they were understanding and creating proportions in the task (the ratio changed from 1 tsp. water: 1 tsp. orange concentrate to 1 cup water: 1 cup orange concentrate, an amount which seemed more appropriate to Betty and the group to distribute among 5 people). The group shared and discussed their ideas more than before when only Fabiola made the decisions. The new kind of interactions that were collectively coordinated had higher levels of what I call alignment among participants as well as more equitably distributed attention, as all students seemed to have equalized positions or similar opportunities to participate and be heard. After this shift, students started using the word “we” more often, thus marking the process as collective. The new interactions provided Betty a new position where she had not only central social, but also mathematical, participation. This shift was supported by both aspects: a) the facilitator’s process of noticing and b) the nature of the task, as it allowed connections to prior experiences—dividing liquid in equal shares for
everyone in a group, a realistic and practical consideration. These circumstances highlight not only the need of having attention and opportunities to participate distributed among all students, but also of paying closer attention to mathematics embedded in the discourse practices of children (Moschkovich, 2002; Ginsburg & Seo, 1999).

b. **Attention to the Task**

In this section, I describe two examples of too much attention given to the task. The first example describes how the facilitator’s attention favored the discourse and interpretations of students who used more conventional forms to describe their ideas and interpret depictions in a task; simultaneously the student who perceived the task in unconventional ways had to acquiesce and simply accept the “right” answers without understanding them. In the second example I describe how the facilitator’s overemphasis on the task placed students in different positions since some understood them more easily and others had a difficult time with the rules. This process hindered the understanding of students who needed the most help and privileged those students who already knew the concepts targeted in the task.

The first example comes from a group comprised of Betty, Fabiola, Candy, Elsita (4th graders), and a female Facilitator, who worked on a task naming fractions. At the top of the page there was a string of numerical representations of fractions with empty boxes above them. Students needed to fill in the boxes to complete the phrase “Fraction Power.” For this, students matched the numbers with a graphic representation of each fraction. Each chart with fractions had a letter, which was used to complete the phrase. I present this example in two parts. In the first part, the facilitator started paying more attention to the student who described the task accurately and ignored the student who described the task with less accuracy. In the second part, I describe how a mismatch of
attention to the depictions of the elements in the task promoted an unresolved conflict that obliged a student to change her answers without understanding.

During the first part, the interactive process started as the facilitator asked the group to share ideas about how to solve the task. Prompted by the facilitator, students provided different explanations about what to do. With different levels of clarity, the students explained how to tackle the task. The facilitator reacted differently to each of the students that spoke. Betty’s explanation received no elaboration from the facilitator. Perhaps it seemed unclear to her. When Betty explained, she mostly relied on gestures to describe her ideas. Her narrative itself was imprecise compared to Fabiola’s. Betty used general terms such as “it,” “this,” “here,” and “there,” which made the content of her message ambiguous, as in the following example:

**Betty:** We need like, this one’s already cuz maybe there’s three right here (points to pie charts); and then the thing is, there’s like three and two (points to numbers, keeps hand there); so, like if there’s three and there’s two right here (points to charts with pen in right hand), then that’s a fraction. We need to write, more like, like if this is one (points to charts), and we need to write, too. So, like if this is one (points to charts with right hand), we need to write it (points to box on top with left).

Fabiola used specific terms such as “fractions,” “letters,” and “up here,” which made her message more direct. And the facilitator elaborated on Fabiola’s utterance immediately after she explained:

**Fabiola:** Yeah, because the fractions are here (points to circles). So, I think we need to put the letter in here (taps empty boxes and numbers), that we see the fraction here (points to circles and holds pen there with right hand), and we put the letter up here (points to boxes with left index).

In a way, Fabiola used gestures to complement her ideas, but Betty used gestures as an important part of her mathematical explanation. Despite these differences, Betty’s explanation helped another peer start understanding the task. Then, the facilitator moved on to identifying fractions only after Fabiola had explained. These alternate ways of providing explanations may depict different levels at which meaning is constructed. For Betty and her peer, having gestures accompanied with a few general terms seemed to help their understanding; for Fabiola, the Facilitator, and Elsita, the use of
more specific terms accompanied by few gestures worked better. These different levels of prolepsis, or ways to capture the audience’s attention (Van Lier, 2004), may indicate different discourse forms or what is “necessary” for some students to understand a task. At the same time, these differences in creating meaning may point to possible conflicts or disalignment in a task based on what the participants focus on and the subsequent actions and positions that they promote. At this point, Fabiola had a powerful position because the facilitator understood her better. Conversely, Betty expressed her ideas “unconventionally,” so the facilitator did not take up and perhaps did not understand her ideas as easily, which led her to ignore Betty’s remark. This fact raises the question whether this process might result from the lack of noticing or understanding what Betty meant and, at the same time, the possibility that the Facilitator might have, with little effort, tried to understand and align with what she meant. Further, these ways of perceiving and acknowledging the different quality of children’s discourses automatically promotes differentiated positions to the students, privileging those who have developed a discourse that is more similar to what the facilitator assumes as correct. This places in disadvantage students who need the highest quality of attention.

The second part of the example describes how, in the same task and group, one student, namely Betty, understood the illustrations in the task differently and the facilitator and peers did not attempt to adapt to this divergent way of noticing the fractions depicted in the illustrations. This fact placed Betty in a powerless position and forced her to change her answers (although correct) without understanding why or how the illustrations represented different or similar fractions.
The group of students struggled with naming the fraction representations. For example, with ½ Fabiola named it “Two out of one,” and Betty agreed with that. The group split into two groups: Candy and Betty saw “R” as ½, and Fabiola and Elsita saw “F” as ½ (See Figure 5). The former pair interpreted the different colors in the chart directly connected to each number, that is, one black sector and two white ones. The Facilitator then told them the correct answer and name for the fraction: one half. Then the group moved on to the next notation, namely “1/3,” and Betty explained and named it correctly. All students and the Facilitator agreed with her. Betty predicted the first word to decode: “I think it’s gonna say FRACTION.” The facilitator, however, asked them not to assume this, but to focus on the task. Students took turns matching the fractions and collectively discussed each answer. Students were confused with 1/8. Illustrations “O” and “M” (see Figure 6, illustrations were adjacent on the actual sheet) portray these similarities. Both are divided into eighths and both have one eighth that is different from the others. If the focus centered on the white sectors, M equals 1/8, and O equals 7/8; but if the focus were on the black sectors, the numbers would be reversed. This duality created conflict, as participants focused their attention on these aspects without reconciling them.

Betty focused her attention on the white sectors of the pie chart (M), and this led her to name it 1/8, which was different from everyone else (and what the task asked) since they focused their attention on the black sectors. Although Betty’s explanation (It’s one covered. It’s one out of eight) resembled her previous description of 1/3 and was endorsed by the facilitator (There’s three [sectors] and there’s only one [sector] covered), Betty’s idea was rejected without her being given a chance to elaborate on her answer. The conflict arose from the difference in how the participants paid attention to features of the illustrations. This difference occurred despite the fact that they were working together. In other words, although they aligned with the task and each other’s actions, they
developed divergent ways of looking at and interpreting the same illustrations, which led to
different ways of making meaning in the task. Both Fabiola and the facilitator corrected Betty,
claiming that there was only one correct answer, “O”, because it completed the phrase. The
emphasis on completing the phrase puzzle overshadowed the possibility of understanding the
representations in divergent, but equally correct ways. It is not clear whether Betty understood the
differences between the two answers, but she accepted what the facilitator determined as correct. In
this process there was no opportunity to explore and resolve differences, so that everyone could
understand or agree. The power was centered on what the task prescribed. The facilitator and most
peers took this perspective, and there was no chance of negotiating otherwise. I believe that if her
idea had been elaborated, this process could have promoted deeper and more meaningful reflection
on the concept of fractions. This process could have caused disalignment from the task, but
alignment with meaning making and a student’s thinking, so that the group’s interactions and
attention would have been placed on what students were thinking, seeing, and understanding about
the concepts embedded in the task, rather than simply completing the task as it was prescribed.

Moreover, Betty’s unconventional way of noticing fractions was confirmed as the group
moved onto naming other fractions (e.g., 3/8). This fact provides evidence that Betty shifted her
attention, switching between the colors of the sectors in order to identify the fractions. The group
had identified “E” as 3/8, and Betty had instead identified it as “U” (see Figure 7). Either pie chart
could have been named 3/8, depending on which color one focused on. It was still not clear to Betty
and the group why this divergence emerged. However, after noticing the difference, Betty changed
her answer without understanding why. Betty knew how the task worked, but not why the “E” chart
with 5 white sectors out of 8 sectors should be called 3/8. Betty, anxious about this difference and
the group pressure marking her errors, stopped participating. Fabiola said that Betty was not paying
enough attention, when Betty was indeed paying attention, as she named the fractions as she saw them and not simply plugged in letters to complete the phrase. Betty’s personal and divergent way of noticing fractions afforded her a powerless position since she had to acquiesce and comply with the group’s and the task’s prescriptions without understanding. This process confused her attempt to understand the mathematical concepts depicted in the illustrations. Her correct interpretations of the representations were only questioned, corrected, and dismissed, but never discussed. Thus, the process of collective alignment that the facilitator promoted did not guarantee that Betty’s way of thinking about fraction representations would be accounted for. Therefore, the contrasting forms of noticing and understanding the elements of the task and how these were deemed as legitimate mediated different positions for the participants. The task had only one “correct” way to interpret fractions, and the facilitator strictly aligned with it. Betty’s ideas and ways of perceiving the fractions were othered and marked as nonmathematical. This process made her become self-conscious of her own understanding and performance, develop self-criticism based on what others said, and decreased her animated way of participating (Bosacki, 2005) in mathematics in her group.

In contrast, Fabiola transitioned from not knowing how to name fractions correctly to being correct, having an appropriate discourse in mathematics, and correcting Betty, etc. Her increasing power in the group was supported by noticing the task and responding to it as expected in the task instructions and by the facilitator. Fabiola’s power increased since it was supported through social interactions in that others embraced her way of seeing and doing mathematics. Legitimized, she had a powerful mathematical position that fed and expanded her frame of action. In fact, the quality of attention provided by the facilitator to Fabiola and Betty contrasts greatly. Thus, this process mediated opposite mathematics performance for these two girls while interacting with the same task and group. Perhaps the facilitator and her peers linked Betty’s “errors” and her unconventional
mathematical discourse to her perceived lack of ability, which promoted the kind of the positions
and support that she had in her group.

The second example comes from a group comprised of Betty, Fabiola, three other girls (3rd
graders), and a female facilitator. The group explained the Counters game to a mother who had
recently joined them. As they played, a conflict arose from the facilitator’s overemphasis on the
game’s rules without appropriately resolving a student’s confusion. Betty (a student with low social
status and grades) seemed confused, as she was randomly moving the counters in an attempt to
imitate her neighbor’s counter placement. When the facilitator noticed Betty’s random moves, she
asked the group to recall the game’s rules, which Fabiola (a student with high social status and
grades) thoroughly explained. The process created different positions for these students, as the
interactions or the facilitator focused on reinforcing procedures rather than understanding the task.

Consequently, the facilitator continued elaborating on what Fabiola described, positioning
her as the student with knowledge of the rules. At the same time, she singled out Betty as someone
who did not know, someone who copied, and someone who was confused. The communicative
process included no attempt to understand the possible reasons why Betty was confused and did not
follow the task’s rules. The interaction simply marked her apparent infraction and reified the game’s
rules. Perhaps the facilitators’ concern about the rules stemmed from their relation to understanding
the probability concepts embedded in the task. One of the facilitators concluded:

Facilitator: Por lo tanto, antes de empezar a jugar, ¿qué es lo más importante siempre que todos los
que van a jugar el juego deben de saber bien? / Therefore, before starting to play, what
is always the most important thing that all players in the game should know well?

Everyone: Las reglas. / The rules.

During the game, Fabiola became the facilitator’s first choice to explain any mathematical
situations in the game. This also encouraged Fabiola, as she was always the first to respond. These
habits afforded her central participation and a powerful position in mathematics by reasoning and explaining how she understood probability. On the contrary, Betty and the rest of the group aligned with the rules in a powerless position, as they simply followed the rules, but with no opportunities to explore the meaning of the game. Betty, under the facilitator’s supervision, focused on the process of taking off from the game board only the counter that matched the sum of the rolled dice. Besides this, there was no clear evidence of her understanding. All players aligned with the task, then, but not with each other. Two parallel but different activities emerged at the table: one promoting the goals of the game—concepts of probability—which seemed lost for Betty and some peers, and another enforced by the facilitator, namely to align to rules without understanding. The approach privileged students who already had an effective understanding of the rules and the task and marginalized those who needed to learn the task and the mathematical concepts themselves.

c. Summary of Attention

The distribution of attention among participants proved crucial in the process of positioning. When students received close attention to their concerns and struggles and these were elaborated and negotiated so that the concepts became clear to them, this process promoted situations in which students were supported, successfully completed the tasks, developed understanding, and manipulated the concepts. Conversely, when the attention centered on the task’s rules, its features, or elements more than on how students were perceiving or understanding the task, the interactive process supported the actions and understanding of those students who aligned with or understood the task better in the first place. At the same time, those students who had difficulty understanding or who perceived or performed the tasks in divergent ways were less likely to have the support from the facilitator and consequently from the group. This situation worsened these students’ opportunities to successfully complete and understand the task, especially the
embedded mathematical concepts. Thus, the quality of attention that students may receive is constrained by how the facilitators and others interpret the participants’ actions. Conversely, the students who received attention were more eager to continue participating and succeed in the task. Thus, the sociopolitical and economic paradox that those who already have are given even more also seems to be replicated in this context. Students who knew more also received more support from the facilitators. Mehan (1979) argues that some types of interactions favor only certain students, those who effectively reply to the initiations presented by the teacher. Students need to adapt to and understand the implicit system created by the teacher. In equitable interactions, a multidirectional process of attention is needed to balance negotiations that capitalize on diversity.

The facilitators play a crucial role in capitalizing on and acknowledging students’ diverse ways of understanding mathematics as well as shifting the interactions that support students who need to learn and understand mathematics better. This process of support is more successful when efforts to engage with students are truly developed in a joint productive activity (Dalton and Tharp, 2002). Joint action and shared attention during problem solving promote student engagement (Barwell, 2003). As the group develops the same focus of attention, they also appropriate mathematical goals (Newman et al., 1989). Additionally, collaboration did not always imply a collective goal or shared attention in this study. A collective goal was disrupted when attention in the group centered on participants’ differences and not on their work on tasks. The process of focusing on students only and singling them out without helping them was an unproductive process of attention, which started by interpreting students’ actions as incorrect without further exploration. Narrow or strict ideas about students’ mathematical discourse may limit the quality of interactions with students and consequently their opportunities to understand and grow from where they are.
3. **Ability or Competence**

Although I have referred to the students in this study as either having low or high social status in the afterschool or low or high mathematics grades in school, I understand ability as an emerging factor during social interactions that is connected to the process of positioning. Ability refers to the different ways in which a participant is perceived as a competent person during interactions or not. In problem solving, this social validation arises through the participants’ exchanges in relation to the values, perceptions, and expectations from others, the task at hand, and oneself. Ability, then, is a variation of attention, a process of noticing a person’s expected or emerging ability to perform. Ability is a collection of skills that are attributed to individuals as part of the specific contexts in which they participate (Gresalfi et al., 2009). Thus, who is “competent” develops through a situated process and looks different across contexts. In this study, I claim that the attribution of ability is intricately connected to the way students are positioned and the opportunities they have to engage in mathematics as they interact with others. In contrast, ability has been traditionally defined as the individual’s aptitude or acquired proficiency to perform a task. Ability is also traditionally associated with general learning, academic performance, or skills assessed through tests. Participants seemed to refer to ability from this perspective. As in previous sections, I present the emergence of ability regarding a) the participants’ ability and b) ability within a particular task.

a. **Participants’ Competence or Ability**

In this section I present two examples that describe how assumptions about participants’ ability promoted the quality of attention and alignment of peers and facilitators and simultaneously affected their positions and actual competence in problem solving. In the first example, a facilitator’s deficit assumptions about a student led her to explain several times the same
task to the student without achieving agreement. The facilitator overlooked this student’s understanding of the task due to her biased perception of the student. In the second example, a student challenged a facilitator by telling him how his assumptions about certain students’ ability marginalized them from participating in mathematical activities.

The first example comes from a group comprised of Fabiola, Betty, Candy, Elsita (4th graders), and two female facilitators. Like the group of boys mentioned in the section above on attention, they decided to work on a game with secret patterns hidden in the following string of letters: SLLSLSSLLSLLSLLSLLSLLSLLSLLSLLS. As they worked, the group split into two groups, each with one facilitator. Candy, across from Betty, worked with Facilitator 2 mostly in Spanish, and the rest with Facilitator 1, speaking mostly in English. During the interaction, Betty was ignored, and Fabiola received a greater level and quality of attention. This process seemed linked to initial assumptions about each student’s ability. Nevertheless, this situation was different at first, as Betty was present during the first explanation. This led her to explain the instructions to Fabiola when she joined the group later. At that time, Betty started to “see” the pattern in the string by overhearing what Candy had developed with her facilitator. They had converted the pattern by counting the number of times each letter repeated along the string, which was: *Uno, uno, dos, dos; uno, uno, dos, dos; uno, uno, dos, dos*. When Betty explained the pattern to Fabiola, she used Candy’s idea. However, the rhythm of the number sequence sounded different when Betty said it in English, and she therefore got confused as she explained. This process led Fabiola and Facilitator 1 to assume that Betty did not understand the task, a situation that started shifting Betty’s position from knower to non-knower.

From that point on, Facilitator 1 began explaining the task to Betty again by using different ideas than Candy’s pattern with numbers. Instead, Facilitator 1 focused on letter pattern. This
contradiction complicated the process for Betty who liked Candy’s idea better and had difficulty adapting to the facilitator’s idea. Fabiola worked with the other facilitator and focused on the task. Betty seemed anxious about not being able to solve the pattern quickly. She was concerned about how quickly Fabiola solved the task. As Facilitator 1 noticed that Fabiola finished, she asked Fabiola to explain her solution to other peers. Facilitator 1 used Fabiola’s sheet to explain the process to Betty. These events positioned Fabiola with social and mathematical central participation: as she spoke, the facilitators listened to what she said and elaborated on it by either suggesting more possible routes or praising her work. Fabiola had the attention and alignment from both facilitators as they ‘saw’ her ability. Betty, on the other hand, had two more explanations from Facilitator 1, but they were about telling Betty what to do without helping her connect what she knew into these other ways of creating patterns. The facilitator looked frustrated, she touched her head, sat back on the chair, rolled her eyes, and breathed deeply, as Betty did not respond to her as she expected. At times Facilitator 1 took breaks from Betty and talked with other students. She made eye contact with the other students, joked, and smiled at them. Here Betty tried to take part in those conversations, but there was no uptake. At times, Betty simply stared and waited in silence. Then when the facilitator returned her attention to Betty, she started rubbing her head again. Facilitator 1 used shapes, alternating circles and triangles, to re-explain the pattern to Betty. She chose the “simplest” way to guarantee that Betty would finally “get it.” This approach was almost condescending: the Facilitator changed her voice, spoke slower, and pointed to each shape, describing step by step. The situation worsened as peers made fun of Betty. At no time did the facilitator attempt to explore how Betty understood the task. The approach was that the facilitator explained and Betty listened. And when Betty made a mistake, the approach was changed again without further negotiation of what Betty meant or knew. The assumption was that Betty did not
understand. Nevertheless, Betty tried to understand Facilitator 1’s approach of alternating shapes by also alternating numbers on a string (e.g., one, one, two, two) and created a pattern of shapes, such as “A, B, A, B, AA, B, A, B, A, BB,” but the facilitator was expecting something simpler, such as “A, B, A, B, A, etc.” Facilitator 1 refuted the example, claiming that “a pattern always has to be the same.” Yet, in fact, Betty’s pattern becomes clearer when one converts it into numbers: “1,1,1,1,2, 1,1,1,1,2.” Thus, it suggests that Betty created a sophisticated pattern that the facilitator could not see given her low expectations of Betty’s ability.

Different opportunities for participants to learn and perform emerged through interactions based on assumptions about the students’ abilities. The pressure was on students to understand the facilitators and not on the facilitators to understand the different ways that students developed. Therefore, those who aligned with the facilitators’ ideas were at an advantage. Fabiola aligned with what the facilitators said or wanted, and she was perceived and constructed as the most able in the group. Whenever she struggled, the facilitators always paid attention to her. Moreover, the facilitators seemed very interested in influencing what Fabiola did. Fabiola’s ideas were supported and validated, as she was asked to explain her ideas to others. On the contrary, Betty seemed drawn to how a peer understood the task instead of the facilitator, and her ability was always perceived as low. Betty had a mathematically and socially powerless position in that she was continuously ignored or scrutinized, but seldom supported. Betty was given procedural, simple explanations that were imposed on her by only one facilitator. Even peers made negative remarks about Betty: “Betty is so crazy, she goes to Special Ed!” “You are so ignorant, Betty!” Therefore, it seems that participants coordinated or aligned their actions better with Fabiola because she aligned with the expected ways to do or be in the group. Perhaps the limitation for the facilitator was her inability to understand Betty’s unconventional way of explaining her ideas. Nevertheless, this possible
limitation was not dealt with, and instead assumptions about students’ ability mediated the
differential quality of alignment and attention given to each student. The different levels of
competence evolved regardless of the accuracy of the actual results, but based rather on initial
assumptions about each student.

The second example occurred in a group comprised of Orlando, Raúl (5th graders), and two
male facilitators. Similarly, facilitators’ assumptions regarding the ability of the participants created
different positions and opportunities for them during problem solving. Here the group created a
poster by debriefing on their process of developing a lemonade recipe and enlarging the quantity for
a party. Taking turns, students recalled and wrote on the poster the ingredients they used in their
original lemonade recipe. Raúl led the conversation and was continuously supported by Facilitator 1
on issues about the amount of the ingredients. Raúl also recalled their concern about converting the
measurements to larger units (ounces to pounds) to describe the amount of sugar more easily. The
fact that Raúl was able to decide and clarify his ideas and be recognized by the facilitator afforded
him a central mathematical and social position. However, Orlando had for the most part peripheral
participation, as he was ignored most of time. Nevertheless, as an active observer, Orlando tried to
get involved by writing down the numbers being mentioned in the conversation and by suggesting
to Raúl ideas to explore and find out the proportional relationship between pounds and ounces, such
as using the ounce glass and the graded plastic container (which they had done the previous day).
Yet the facilitators’ response to Orlando to be careful not to break the glass he was holding failed to
address his ideas. The process of crafting the poster had clear, fixed roles for each student: Raúl did
the mathematics and Orlando the decorations.

The conversation in English centered on the conversion from ounces to pounds. During this
time Orlando was also ignored. Then Facilitator 1 and Raúl debriefed and clarified ideas a second
time, but now in Spanish. As Orlando again unsuccessfully attempted to join the conversation and the mathematical interaction, he shifted to making fun of what was being said in the conversation with unrelated numbers or ideas. This behavior, which could possibly be described as misbehavior, seemed Orlando’s way to evidence the need of attention from the facilitators. Orlando’s productive actions related to mathematics were overlooked, and his nonmathematical actions were corrected and singled out as unproductive. Orlando’s attention concerned Raúl also. What Raúl obtained as well as what Orlando himself did not receive from the interaction was noticeable to Orlando, but not to the facilitators. Facilitator 1 scaffolded Raúl’s process for converting ounces to pounds. This process was relevant to both Facilitator 1 (F1) and to the other Facilitator. Thus, Orlando—from his perspective in a powerless position—exposed how the group’s interactions around problem solving privileged Raúl and marginalized him and the other two peers, who were not “good” at mathematics and had fewer opportunities to participate in mathematics:

F1: (to Raúl) Uh-huh! Entonces 2 libras son 32 onzas. Escríbelo ahí. / Then 2 pounds are 32 ounces. Write it down there.

Orlando: (whispers as Facilitator 1 talks to Raúl) Raúl siempre hace todo. (talks out loud) Raúl siempre tiene que hacer todo lo difícil. / Raúl always gets to do everything. Raúl always gets to do the hard things.

F1: (to Orlando) ¿Por qué dices eso? / Why do you say that?

Orlando: (points to Raúl’s work) Porque está haciendo matemáticas. / Because he is doing mathematics!

F1: Pero tú también puedes. / But you can do that, too.

Orlando: No, no puedo. / No, I can’t.

F1: Sí, tú también puedes nada más que… / Yes, you can, too; but...

Orlando: No, no puedo porque siempre le hacen a él. / No, I can’t because they always make him do it.

F1: Pero tú lo hiciste en clase conmigo la otra vez, ¿te acuerdas? / But you did it with me in class the other day, ¿Remember?
Orlando: (writes) No, pero aquí siempre le hacen a él, no a mí, ni a Antonio, ni a Juan. / No, but here they always make him do it, not me, or Antonio, or Juan.

F1: (to Orlando) No, también ustedes me ayudan mucho. Ustedes comparten todo lo mismo, pero está bien. (to Raúl) ¿Sí entiendes Raúl que necesitamos dos libras y ¼ de una libra para hacer las onzas completamente? ¿Sí? ¿Y puedes explicarme cómo y por qué? / No, you guys also help me a lot. You guys share all same things, but it’s OK. (to Raúl) You do understand, Raúl, that we need 2 pounds and ¼ of 1 pound to have all the ounces? Yes? And can you explain to me how and why?

The relevant mathematical event of the interaction to Facilitator 1 and Raúl was to decipher the ratio of ounces to pounds in 36 ounces. Raúl was not doing it all alone, for he had a reifying, always present, supportive system of performance, a mathematical and social privilege that Orlando noticed and exposed. At the same time, this system excluded and took away opportunities from Orlando and others like him, who were assumed not to be good at mathematics. Orlando also mentioned how this process had contributed to his developing a distant relationship to mathematics and had blocked him from chances to engage with complex mathematical concepts. He argued that his perception of his lack of mathematical ability had been informed by similar scenarios where the complex mathematical tasks were given (by authoritative figures) to students like Raúl (assumed to be good at mathematics), and had led Orlando to self-identify as a non-doer of mathematics.

Therefore, the mathematical and social positions, co-constructed during interactions, worked as sorting mechanisms rooted in social actions and assumptions about participants’ performance. At the same time, it gave students opportunities to compare their knowledge and mathematical status in the group. Then, some students had the opportunity to learn more mathematics and feel more capable than others who did not. This process provided mathematical voices to students whose actions were perceived as mathematical (Raúl) and silenced those whose actions were not deemed so (Orlando). The main point here is that although these events might have seemed nuanced or
perhaps invisible to some of the participants in the group, they were not to others, such as Orlando, who was in the midst of it as the object of exclusion and marginalization during problem solving.

Another aspect of this argument is Orlando’s exaggeration, using terms such as “always.” Thus his statement could overgeneralize the situation. I further discuss this point later on in the section on students’ positions and participation over time. Nonetheless, I point out the fact that Orlando had ready the names of other students who were excluded as he was. This demonstrates either a recurrent process of observation or perhaps a socialized process that informed Orlando. Thus a previously realized process that had emerged was supported by the interactions at that moment. Moreover, Orlando insisted in many different ways to show that he was interested in following the mathematical conversation, and, as he was rejected, he moved onto unproductive behaviors to gain attention. There was an explicit process of ignoring Orlando which contrasted with the explicit process of overvaluing Raúl’s actions, a situation that Orlando interpreted as directly linked to student ability. In all of this, Raúl remained distant from the discussion.

b. Ability or Competence in a Task

In this section I present two examples and describe how the structure of a task promoted different levels of student participation in the same group. This, combined with social interactions, mediated the definition of students’ competence in the task, their opportunities to participate, and their positions in the group. In the first example, the facilitator provided students differentiated opportunities to explore probability concepts based on the assumptions he made about each student. In the second example, a group of facilitators attempted to support socially a student with low performance after the student confessed to them her lack of confidence in her own ability. The facilitators’ support failed, as they only supported the student socially but not through reflective processes on mathematical concepts.
The first example comes from a group comprised of Raúl, Orlando, Antonio, Mario (4th graders), and two male facilitators, who worked on a task called “El Maga’s Hat.” In this activity, students took turns pulling out one plastic tile at a time from a bag that contained a total of 16 tiles (7 reds, 5 blues, and 4 yellows). The purpose was to explore and predict probability patterns by recording the outcomes, analyzing the bag content, and making connections between the two. As students pulled tiles out and recorded them on a sheet, they cheered up every time their favorite color came out. The activity took about 40 minutes. Then they totaled the number of times per color, but some students noticed mismatches in their results and decided to review their sums.

Orlando worked on checking his sums. He noticed a pattern and said “red won because they came out the most.” Facilitator 1 followed this idea and continued asking students for predictions. They all agreed with Orlando. Then the group explored the content of the bag to reflect on the number of tiles by color and the patterns that they had recorded. Raúl predicted a greater number of red tiles. With tiles on the table, Facilitator 1 asked students to compare differences and make connections to probability. Students noticed that red had the highest number and that there were two more red tiles than blue. Orlando went beyond and removed the extra red and blue blocks and made all colors of even heights (see Figure 8).

Figure 8: Tiles in El Maga’s hat

Orlando’s idea prompted Facilitator 1 to further explore probability along a similar pathway. This also helped Raúl find a probability contradiction since their count of yellow tiles was greater
than the blue tiles, when in fact they had fewer yellow tiles in the bag. As all students agreed with this, they started rechecking their sums to make sure they were correct. Although this fact was an important element to make sense of the task, the same process led to a procedural review of the previous sums and not of the concepts. Here, Facilitator 1 relied during the entire process on Raúl’s results, which mismatched with Orlando’s. This situation positioned Raúl as the competent student in the group, and other students’ results were subordinated to Raúl’s findings. This powerful position afforded Raúl a sense of confidence in his skills and ability during the problem-solving task. Orlando seemed very concerned about having the correct sums, which made him revise his results by comparing them with Raúl’s. Facilitator 1 elaborated on Orlando’s original idea of comparing the tiles, but the facilitator worked through this process only with Raúl. Both students were at the table, but there was no direct attempt to involve Orlando, who was concerned about having perfect sums. The main goal of the task was to explore probability patterns, but it was a privilege that only Raúl had. As he only had the opportunity to clarify issue about the yellow tiles.

In a way, Facilitator 1 privileged and nurtured Raúl’s initial competence. He not only spent more time with but also instructed Raúl better and let him explore deeper levels of mathematics. Meanwhile, Orlando simply dealt with basic mathematics, such as adding. In short, the purpose of the task was achieved for Raúl, but not for Orlando. After the task was completed, there were different levels of understanding mediated through interactions and assumptions. The process contributed to Orlando’s accumulated gaps of knowledge and Raúl’s accumulated knowledge. Thus, levels of initial performance in a task may influence the way a student is perceived and also mediate the sequential supporting actions for those who demonstrate competence. Those with weak competence may be simply ignored or left to do basic mathematical operations. Although Orlando showed great potential for success through his contribution of separating the tiles, this was
overshadowed by the facilitator’s overt support to more competent students in the group (Raúl), which in turn promoted Orlando’s anxiety about his own competence. Orlando’s powerful idea was not elaborated with him. Rather, it was “stolen” from him, and instead Raúl elaborated on and benefited from it, legitimized by the facilitator’s actions and assumptions about students’ competence based on the task constraints and their initial performance.

The second example comes from the group comprised of Fabiola, Betty (5th graders), and three female facilitators. They played a game rolling dice and adding results algebraically, which at times provided negative numbers. They rolled dice of two different colors and always needed to subtract the number that came up on the green die from the number on the yellow die. The concept of negative numbers seemed new to them. Both students had difficulty remembering how to play the game and clearly understand how to subtract the numbers (which in a way resembled algebraic addition) and get either negative or positive results. The group started playing and, as they played, received feedback from facilitators. This evolved into creating their own board game (with numbers -5 through 5) and placing the 11 counters each participant had on that board. All took turns rolling dice, and students figured out results. For example, when the rolled dice read “2-3,” Betty said “it’s nothing!” Fabiola guessed: “5, negative 5, one,” and then said “zero.” Since both students struggled with the task, the facilitators created and used a number line as a tool to explain how to obtain correct responses. Yet the number line was placed next to Fabiola, and only she used it. Eventually, the process evolved into Fabiola answering all problems and Betty reading the rolled numbers. The access to this tool, combined with the facilitators’ assistance, afforded Fabiola a quick and firm grasp of how to operate in this context. Betty responded correctly only when the results were positive numbers, but not with negative numbers.
This situation promoted different student performances and positions. Betty could not operate the results with negative numbers, a situation which made her anxious about her performance. Fabiola often pressured and made fun of her. Betty linked this pressure and her not understanding to ‘not being smart.’ Still, Fabiola also made mistakes when negative numbers were involved, so both students seemed at a similar stage, but with differentiated access to resources (number line), adult support, and social pressure. To Fabiola it was a learning supportive process. For Betty, it was an anxious social context, where she was continuously tested. At one point, Fabiola left momentarily, and the facilitators worked more closely with Betty and explored the different combinations of numbers from the dice rolls in relation to probability. Nonetheless, Betty resisted the process by confessing to the facilitators concerns about her ability:

F 1: (Facilitator 1) Whenever you subtract a number from the same number, like fifty-four minus fifty-four, siempre te dará lo mismo [it will always give you the same], which is zero.


F 1: Yes, you are. Nomás te da huevonada! [It’s just laziness!]

Betty: (looks down, puts hands on head and starts raising hands as if yawning)

After Betty confessed her perceived lack of competence, the facilitators started supporting Betty more carefully with explanations and encouraging her positively. Although Facilitator 1 used a negative term, in her continuing actions she tried to convince Betty that the problem was not a matter of intelligence, but of personal effort. The new process mediated another level of attention to and alignment with Betty on the part of the facilitators. However, the kind of interactions that evolved resembled an IRE pattern and not a conversation helping Betty understand the concepts of negative numbers. Instead, the interaction or instruction was to provide explanations about the procedures to complete the task without reflecting on the mathematical concepts. When Fabiola returned, though the facilitators supported Betty differently and positioned her powerfully, Betty’s
physical posture changed immediately, sitting up ready to work. Yet Betty had not yet understood the mathematical concepts, and although Fabiola made some mistakes, she turned the interaction pattern around and again started correcting and answering for Betty. Thus, Fabiola’s capital of knowledge and assurance again took over, minimizing Betty’s opportunities to explore problem solving, despite the facilitators’ intended support.

This situation describes how facilitators were more willing to pay attention to and align with Betty’s actions after they understood her situation and conflicts better. Facilitators knew that their actions could improve and perhaps equalize the collaborative process by providing more support to the student with low mathematical status or position. However, their support to Betty failed, in that the quality of the interactions around meaning making remained the same, not supporting Betty’s understanding of the concept. Despite the mathematics support given to Betty by explaining to her the procedures to complete the task and the social support by being “nice” to Betty by responding to her when she talked and having her take a more central role in the group’s actions, these steps did not improve her mathematical competence. This situation widens our understanding of what social support and support for performance mean. Perhaps if the process had focused on supporting what Betty knew—or not—and helped her build from there and uncover the connections or better understandings of the concept of negative numbers, this process could have complemented the social support that the facilitators showed for Betty. Perhaps a process similar to that provided to Fabiola, in which they noticed what she knew—or not—and supported her accordingly, would have been more helpful. Trust, support, challenge, and coordinated actions helped Fabiola understand negative numbers. Thus, the kinds of attention and alignment need to be situated and adapted to students’ needs so that they navigate the situation successfully as they develop their own mathematical understandings within this supportive system (Cohen, 2000; Gresalfi et al., 2009).
Thus social processes are relevant in how people come and work together noticing and listening to each other and respond with actions that align and are pertinent to others’ actions in the same task. Social processes in mathematics problem solving activity then imply a “walking” together (socially) trying to achieve the goal of understanding the mathematical concepts and the solutions of the problematic situation, the object of the interaction. Here, social processes also include giving opportunities to all participants, not equally but equitably, adjusting the support for everyone to access and understand the socialized ideas (without assuming their understanding but rather negotiating them), so that these ideas about the mathematical concepts are shared and collectively constructed goals, actions that help build one’s understanding of the mathematical ideas and solutions. These ideas and forms of interaction are similar to what has been proposed by approaches such as Complex Instruction (Cohen & Lotan, 1997). Thus, participants’ honed attention and alignment to others are crucial for mediating greater levels of distributed mathematical competence for all participants. The productive process in this group started after facilitators noticed Betty’s complex situation not only about her struggle with the current concept but her overall perspective of herself as a non doer of mathematics (not smart) and facilitators developed empathy with her and tried to support her socially. The productivity was lost when the social interactions did not support Betty’s mathematical needs and understanding about the task.

c. Summary of Ability

Ability or the development of competence corresponds to a socially mediated process between an individual and the resources available in an environment and how these resources are made available and used. It is at this intersection that the social environment facilitates and qualifies the individual’s level of ability or competence. During problem solving, the students’ initial mistakes or accuracies seemed to pave the way for differentiated levels of support or attention
and alignment that the facilitators and the group provided each student. Assumptions about students influenced subsequent actions, especially those of the facilitators. These different levels of support were obvious to students, especially to those who had poor support or were deemed to have low competence. This process often prevented them from learning, understanding, or performing the task. Their potential was frequently undervalued, as the low assumptions obstructed possibilities to see these students’ potential. Assumptions led facilitators’ and peers’ interactions to support those students who seemed to know more. The support from facilitators and peers for these students—working with them, being attentive to their needs and questions, and correspondingly responding to and supporting their performance around the task—appeared natural and a matter of course. With students assumed to have low performance, the supportive process was almost seen as a burden; the low expectations often promoted procedural rather than conceptual knowledge, and the tasks were watered down. Often these students’ ideas were not attended or listened to carefully, which led to mismatches during interactions. As some of the examples provided here portray, facilitators were concerned about all students and when some of these students shared their difficulties with the facilitators, facilitators empathized with and cared for these students. Facilitators tried to provide a supportive system that was nevertheless not always successful since established forms of interaction resurfaced. Consequently, a conscious process to alter these trends is necessary, and this should include a process beyond being “nice” to students, but rather paying attention to their understanding and promoting situations in which they can reflect and act according to what they know. In other words, students’ resources and current understanding serve to build greater ones. It is interesting that this process is almost automatically done with students who are assumed to know more, but once the idea that a student does not know is in place, it complicates the process and subsequent actions try to “fix” the problem without paying attention to and aligning with the student.
C. **What Role Does Language Play in Positioning?**

This section describes how language, Spanish or English, was used by participants in the process of positioning themselves and others. Although the three factors previously explained (attention, alignment and ability) were always present, at times participants made use of language in specific ways related to the power differential of languages and used codeswitching as a tool to mark and position themselves and others in mathematics. Before starting a task, the facilitators asked students to pick a version of the task to read—either in English or Spanish—and the English version was the most frequent choice. However, when these options were not presented as choices, students used either language. Some groups preferred to use one language more, but English was the most frequent choice across groups and time. Spanish usage had a peak during the family recipes unit. There were a couple of sessions when almost all groups interacted in Spanish only, namely when the mothers played the Counters game in the groups and when the mothers led the creation of a recipe. Yet there were other times when, despite their presence, English was spoken often. For the analysis of language and positioning, I explored times when participants codeswitched within the same conversation, which appeared to have a situated purpose related to the process of positioning. In this process, I identified four purposes in codeswitching: 1) to mark differences or note incompetence, 2) to gain authority, 3) to prevent positioning, and 4) to differentiate language status and positions. In the following, I present each of these patterns in the same order.

1. **Language Marking Differences and Mathematical Ability**

Participants seemed to use language codeswitching as a way to mark special situations, moments, or actions during social interaction. That is, when they were speaking in one language, at times they shifted to the other. As I explored these patterns, I found that they indexed
differences, not merely errors. The purpose was apparently to have other participants notice something specific to be addressed. The further exploration of these marking strategies discovered four forms or language functions: to patronize, to other, to correct, and to clarify or elaborate. These forms served as tools for participants to develop attention, alignment, and/or noticed ability. I define these different language functions as follows:

- **To Patronize**: To assume an air of superiority towards; to treat or speak about (e.g., a person) condescendingly, or with apparent indulgence or kindness.
- **To Other**: To describe someone as different or conceptualize her/him as intrinsically different from oneself or one’s group.
- **To Correct**: To set right or amend (a thing); to substitute what is correct for an error; to point out or mark an error in order to amend it.
- **To Clarify or Elaborate**: To make clear (an obscure idea, concept) and to work out in detail; to give closure or completeness to a work, an idea, an operation, etc.

The number of language shifts around positioning events were recorded and categorized by function and language. Based on the totals by category, I graded their relevance using a scale from 0-3. Each number in the table (see Table III) represents a specific amount of use by participants of a function in a specific language (i.e., zero, = absence; one = little use; two = some use; and three = significant use). When participants spoke in English and wanted to mark differences, at times they did so by shifting to Spanish or vice-versa. Therefore, the instances of codeswitching to Spanish are greater than to English, as participants spoke English most of the time.

<table>
<thead>
<tr>
<th>Language</th>
<th>Patronize</th>
<th>Other</th>
<th>Correct</th>
<th>Clarify</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spanish</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>English</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Key: 0= none, 1= little use, 2= somewhat, 3 = significant use

Participants used codeswitching to mark differences, often with the attempt to clarify or elaborate on an issue or mathematical concept and prevent further confusion. This function portrays proactive actions at negotiating differences during problem solving. To a similar degree,
participants codeswitched to mark a difference and correct what seemed wrong. These two forms of noticing differences seemed related to the process of negotiating mathematical ideas or procedures while problem solving. The apparent purpose was to promote alignment and negotiation of the actions and operations in the group. Nevertheless, these negotiations were not always successful and therefore affected both the positioning process of participants and the process of marking differences. Some functions directly addressed individuals or their actions, which led to patronizing or othering processes. The othering process developed through codeswitching to either language with the goal of marking others’ differences and separating them from oneself, thus creating distance among participants. Finally, participants patronized others by using Spanish only. The use of a low-status language (Spanish) within a discussion in a high-status language (English) represents a process of condescending to others to mark their differences by doing so in their language, thereby making sure that the message gets across clearly. This kind of alignment using the same language of the person who is patronized functioned to separate that person from the group. Consequently, these language functions not only marked differences, but also provided positions through language switching. Table IV portrays patterns in each of these functions and how they marked differences.

**TABLE IV**

<table>
<thead>
<tr>
<th>Function</th>
<th>Spanish</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>To Patronize by</strong></td>
<td>-‘Nicely’ expressing a lack of ability, pity, pejorative description, or boredom about or dismissing others’ actions/ideas. E.g., a) Entiendes, Betty? /Do you understand, Betty? and b) ¡Pues te hablo en español! /Well then, I’ll speak to you in Spanish!</td>
<td>Not found</td>
</tr>
<tr>
<td><strong>To Other by</strong></td>
<td>-Marking difference in ability of others from the rest of group. -Describing different characteristics of others through cultural metaphors. E.g., a) Con la tortuga de Betty…/ Betty is like a turtle and b) Se te nota que te cargas la huevonada. / It’s obvious how lazy you are.</td>
<td>-Marking someone as different using different adjectives or calling names. -Code-switching to English without elaborating on others’ remark in Spanish. E.g., a) Betty is so crazy, she goes to Special Ed! and b) Cheater!</td>
</tr>
</tbody>
</table>
In addition to the issue of patronizing, two main differences emerged from the patterns presented above. Firstly, the process of clarifying or elaborating evolved in both languages and was used to guide collaboration during mathematics problem solving. However, the clarifications varied depending on the language. In Spanish, for example, codeswitching was used to the check on the steps or processes in problem solving. In English the process seemed connected to mathematical elements such as naming numbers or operating with them. Thus participants switched to Spanish while otherwise performing in English in order to check the problem solving and agree on the process. On the other hand, when performing in Spanish, they switched to English in order to ponder details or finalize the results. Secondly, the use of Spanish to correct seemed to be more explicit and directive than when it was in English, wherein the process was promoted through questions and/or indirect prompts. Thus, again, codeswitching to Spanish was used for explicitly

| To Correct by | -Pointing out misbehavior, lack of engagement.  
- Presenting questions to draw attention to an issue in the task.  
- Providing clear directions accompanied by physical demonstration.  
E.g., a) Y pones éste acá. / *And you put this one here*, b) Pon atención. / *Pay attention*, and c) Así mira. / *Look, like this*.  
- Adding missing points to ideas/claims.  
- Presenting suggestions or remarks that indirectly correct ideas or claims.  
- Asking questions on emerging issues.  
E.g., a) Wait! I didn’t get that, b) Wait, so we need a…, and c) Why would you put six in the orange juice concentrate? |
|-------------------------------|
| To Clarify/ Elaborate by | - Exploring more aspects of an issue.  
- Providing brief definitions.  
- Providing open options for action.  
- Soliciting continuation of conversation.  
- Emphasizing something in discussion.  
- Double-checking future actions or steps.  
- Bringing up features of the task.  
- Indicating modes to do the task.  
- Checking for agreement.  
- Making sure the others understand.  
E.g., a) Lemon juice, verdad? / *right?*, b) A ver, ¿cuál va a ser el número tres? / *Let’s see, which will be number three?*, and c) Whenever you subtract a number from the same number, like fifty-four minus fifty-four, siempre te va a dar lo mismo / *It will always give you the same*.  
- Marking emotions related to the process in the task or interactions.  
- Concluding or summarizing the processes of mathematical actions/operations.  
- Naming a mathematical concept, term, or number.  
E.g., a) Yay! ¡Ya gané!/ *I won already!*, b) No ponemos cinco en cada uno? / *Don’t we put five in each?* So they are equal?, and c) No va a seguir haciendo / *It’s not going to keep doing*: one, two, one, two, one, two! Es / *It’s*: one, two, two, one, one, two, two, one, two, two, one…?? Something like that. No va a ser / *It’s not going*: one, one, one, one… |
correcting and guiding others, and corrections in English promoted more reflective processes. Codeswitching to each language, then, had nuanced uses during problem solving. The use of Spanish did not always intend to ameliorate differences in mathematical understanding. On the contrary, at times Spanish usage simply marked as different those students who either spoke mostly in Spanish or did not understand the task. Therefore, Spanish use within an English-speaking ecology was not always helpful, as the following conversation illustrates:

**F2:** (to Orlando) Well, you wanna talk about it?—about the questions—I can help you, talk about it, to think about some questions.

**Orlando:** (almost whispering) I don’t want to talk about it in English.

**Raúl:** He do not know that much English.

**F2:** ¡Pues te hablo en español! / Well then, I’ll speak to you in Spanish!

Facilitator 2 (F2) offered help to Orlando and also the possibility of doing it in Spanish, but he switched back to English and worked with Raúl instead. Confused with the task, Orlando could not follow the group’s actions, refused to use English, and was ignored. The process identified him as someone with limited English skills. Codeswitching served to other and patronize Orlando. The theme of language served as an axis to mark differences and ability in the group.

2. **Language Used to Gain Mathematical Authority**

Students used both languages in mathematical discussions around problem solving to understand and solve the tasks by arguing with and challenging each other. For this, I explored patterns of language beyond single statements to extended language use during a problem-solving activity. Consequently, I observed for the frequency of participants’ language use by recording who spoke to whom and in what language during one-minute intervals. I analyzed sessions where interactions evolved fluently in both languages. Results indicated students used English in relation to gaining authority in their claims during problem solving. I present one example that I used earlier
in the section on attention. Here Betty (a student with low status and grades) and Fabiola (a student with high status and grades) created orange juice mixtures in their group. The analysis centered on Betty’s interaction during 25 minutes of group discussion. All participants were bilingual, but she used the most Spanish in the group. Betty was initially ignored, but when the facilitator acknowledged her idea, she started participating more and her levels of English increased. This discussion evolved in various stages, which are described on the horizontal axis of Figure 9. The dotted line represents Betty’s use of Spanish and the solid line, English.

**Figure 9**: Betty’s leadership related to language use

Arrow 1 shows the moment when Facilitator 1 acknowledged Betty’s challenge. This situation or action initiated a new position for Betty’s participation in the group. Previously she was ignored during the evolution of the mathematical task since Fabiola was leading the process and all the attention of the facilitators supported Fabiola. While Betty was excluded she spoke mostly in Spanish. But after point 1, Betty’s use of English increased as she started articulating her ideas that would support her claims. She switched to Spanish in order to elaborate her mathematical reasons.
(e.g., Porque mira, él hizo una y esto es poquito y no va alcanzar para cinco / Because, look! he made one and this is only a little bit and it is not going to be enough for five people.) and she made use of English to correct Fabiola’s approach and support her claim (e.g., But we have to put more). She used the most English when the group negotiated the proportions of the second mixture and also decided who could develop the different steps to complete the mixture.

At point 2, Betty, who was confused with the term “to pour,” spoke in English to justify herself. At point 3, Betty spoke in English to lead the distribution of juice among the members of the group. She used English to emphasize the relevance of her ideas and to coordinate the process. As Betty gained authority in her mathematical position, she seemed to have more occasions to use English. Thus, Betty moved fluidly between the two languages and used them strategically. Likewise, the rest of the group also seemed to make similar use of language. When students picked roles in the task, they also used more English. Once the roles were established, the group conversed informally and used more Spanish. Finally, in the last segment, Betty made a mistake measuring the juice for the mixture. The group criticized her, marking differences by othering and patronizing her in Spanish. This situation returned the leading role to Fabiola, who immediately used more English.

Therefore, when students attempted to gain authority—especially Fabiola and Betty—they made greater use of English. At the same time, Spanish seemed to be used either for social purposes or to patronize Betty (who had used more Spanish to explain herself). Students’ language use resembles normalization processes, since linguistic power was connected to English (Cashman, 2005; Garafanga, 2005; Lippi-Green, 1997; Soto, 2002; Tejeda, Espinoza, and Gutiérrez, 2003). They used the power associated to each language (English or Spanish) as strategic means to either gain authority and/or undermine other’s actions, which also mediated different the mathematical position for themselves and others during problem solving in small groups.
In addition to using language to mark differences and mediate power or authority, students used both languages fluently while doing mathematics. Nonetheless, certain students used Spanish to scaffold their mathematical actions only in private situations. Thus, these actions implied a way of positioning oneself in relation to others in the group while doing mathematics.

Some students seemed to have memorized mathematical facts, perhaps in school, so they used only English when referring to them. For example one could hear “five times five is twenty-five” within a conversation in Spanish. Nevertheless, students who used more Spanish while doing mathematics often shifted to English during collective interactions. Therefore, there seemed to be more room to practice mathematical skills in English across groups, and the use of Spanish was relegated little by little to situations in which students spoke to themselves. One example comes from Betty. Her group was working on finding patterns. Betty found the task complicated, but she started understanding it when she overheard a student in another group describe the pattern in Spanish in a rhythmic way (Uno, uno, dos, dos). Since her group spoke in English, when Betty explained her ideas, she used English but became confused as she translated the pattern. This led the facilitator to assume that Betty did not understand and to provide extensive procedural explanations to her. At no moment did the facilitator try to build off of what Betty knew. Instead, the facilitator fed the information to Betty, who received low quality of attention from her facilitator. When Betty noticed that no one heard her, she used Spanish to help herself remember the pattern. Eventually, she solved the task. Betty’s peers were bilingual, but they spoke in English. Betty could have used Spanish to speak about mathematics with her group, but instead she hid it.

The other example comes from Orlando. The example has two parts. In the beginning of the afterschool, Orlando (a 3rd grader) participated in a group playing the Counters Game with a female
peer and two male facilitators. They spoke mostly in English. And although Orlando participated in English fluently, at times he codeswitched to Spanish, helping himself operate some numbers. Once he felt ready, he went back to playing and speaking in English. The second part developed later on when he was in 5th grade. He worked in a group with three boys and two facilitators. They played “El Maga’s Hat.” Here they also interacted in English, and at the end of the task they needed to add extensive amounts of numbers. Orlando added in English, but when no one noticed, he counted in Spanish. If others came and talked to him, he shifted back to English. All participants around the table were bilingual, yet English and Spanish seemed to have different statuses to be used when operating mathematically. I wonder whether school demands had made him feel the need to use English only. Regardless of the reason, his language choices differed from earlier. His public self-speech guiding his thinking using Spanish was subordinated over time. Earlier he positioned himself publically as a Spanish speaker. Later on, he acted as if this language had to be hidden. These examples portray a process of becoming self-conscious about using Spanish when performing in mathematics. This situation surprised me, since all students seemed to use either language freely. Perhaps there is pressure for students to use more English in mathematics.

4. **Differentiated Status of Language**

Students seemed to use language-associated power in more direct forms to position one another. First, Spanish was considered a lower status language. Orlando expressed this during the last interview where he identified some toys as either English or Spanish speakers. He selected a greater number in the English group (5 to 3). He eventually chose two from the English group and defined them as bilingual. He assigned jobs linked to language, as described in the excerpt below:

**Interviewer:** ¿Y de qué trabajan? / And what’s their job?

**Orlando:** Éstos/ These [the bilingual group] en / in Mc Donald’s.

**Interviewer:** ¿Y éstos? / And these [pointing at the English group]?
Orlando: En Inglés sin Barreras. / In English without Borders.
Interviewer: ¿Y éstos? / And those ones [Spanish group]?
Orlando: They are cookers.
Interviewer: ¿En la cocina? ¿Cocineros? / In the kitchen? Cooks?
Orlando: (nods)
Interviewer: ¿Y por qué? / And why?
Orlando: I don’t know.

After this, Orlando switched jobs between the bilingual and English group, arguing that people in “English without Borders” (now the bilingual crowd) would teach each group the other language they do not speak. It is important to clarify that the McDonald’s restaurant in the neighborhood is a nice place, a restaurant where one is more likely to speak with and find English-speaking people (than in a Mexican restaurant, for example). Orlando seemed aware of power differential structures and how people that speak one language or the other may have different positions in society as well. Jackson (2009) describes that children are not only socialized to use language, but also that the socialization process evolves through language. Based on these ideas expressed in Spanish by Orlando at the end of the program, it is my impression that Orlando’s personal way of scaffolding his performance in mathematics was compromised over the years to the point that by making public use of it, it could harm his position rather than help him. Thus, doing mathematics publicly in a second language became the legitimized way, perhaps with weaker potential, but with the expected, “correct” form.

Besides perceptions of the low status of Spanish use, I also tracked the use of language in all the analyzed sessions by each student. When a student used only one language most of the time, then I counted that session as one language, and when both languages were to a similar degree, then I counted that session for both languages. Results show that all students used less Spanish than English. All students used English over 60% of the time. Spanish was used as follows: Orlando
37%, Raúl 22%, Betty 32% and Fabiola 27%. Nevertheless, from students’ perspectives, Betty and Orlando—both with observed low social status and grades and with higher percentages of powerless positions over time in this study—were constructed as Spanish dominant speakers and as less fluent English speakers. This might relate to the quality of their fluency while speaking. However, the most language differences that I located were a few instances when they struggled understanding terms. Otherwise, all students seemed fluent in both languages. The analysis of the different interactions indicated that as students worked together they paid more attention to errors in English than in Spanish. Thus, Orlando and Betty’s confusions using English became prominent, raising concerns about their English fluency and confirming their Spanish dominance. Orlando and Betty were the only two students out of the fifteen who self-identified as balanced bilinguals, another as Spanish dominant, and the rest as English dominant. All students mentioned using both languages in and out of school settings. All had parents who mostly speak Spanish at home and siblings with whom they prefer to use English. Thus, the fact of speaking both languages in nearly similar ways and percentages in the afterschool was not enough for these students to perceive themselves as bilingual. In a way, they all embodied their bilingualism, but they perceived it differently and almost independently from their actions, but rather closely connected to their quality of interactions. These interactions mediated cumulative levels of positions and perceptions of their language fluency and identity, all of which were parallel to language ideological systems of control and power (Bourdieu, 1977; Cashman, 2006; Fairclough, 1989; Lippi-Green, 1997).

5. **Summary of the Role of Language in Positioning**

In this section, I presented four major patterns of the role of language in positioning. First, codeswitching was used to mark differences among participants. Language, at times, was used to clarify, correct, other, and/or patronize one another. Thus, the positioning process emerged
through speech acts, but specifically the shifting of languages within one conversation seemed to
serve as a way to evidence difference in the process of positioning. Second, English seemed to be
utilized as a process to “gain” authority during mathematical discussions. Again, the power
associated with language was accessed by the participants to validate their claims and promote
others’ attention to what they said. Third, although Spanish was used to support one’s mathematical
performance through self-speech, it seemed gradually subordinated during public or collective
actions, even when all participants were bilingual. However, Spanish was still accessed when no
one else noticed. This process indexed a private, powerless position of Spanish use. Lastly, the
discursive use of language mismatched how students self-identified with a language, a fact that was
influenced by lines of power.

These power differential aspects during communicative processes seem linked to language
ideologies that maintain hegemonic structures (Fairclough, 1989; Lippi-Green, 1997). Issues of
power in the sociocultural, political context are still affecting children’s lived realities (Soto, 2002).
The power structures identified by these students appear to play a role in choosing a language,
which automatically seems connected to choosing a position of power or not (Delpit, 1988; Nieto,
1994). These processes call for a careful consideration of issues of language that, in turn, may
transfer into an influential pattern affecting who develops powerless positions and low performance
in mathematics and perhaps drop outs among the Latina/o student population.

D. **Does Positioning Change Over Time?**

This last section of findings includes a summary of students’ overall positioning and
participation patterns during their work in the afterschool. I describe their trajectories in three parts:
1) general group participation of students; 2) patterns of positioning and participation over time in
relation to the tasks, groups, and language use by comparing and contrasting the experiences of
students with low (LSG) and with high social status and grades (HSG); and 3) students’ self perception of their opportunities and personal relation with mathematics.

1. **General Group Participation Trends of Student Cases**

The students selected as cases participated in different types of small groups. They freely chose the groups and facilitators they worked with, and over time their most frequent choice was to work in the group with same gender peers. This means that Fabiola and Betty as well as Raúl and Orlando worked most of the time in the same group of peers, respectively. The percentage of time shared in the same group with respect to their overall attendance to the afterschool was: Fabiola 55%, Betty 51%, Raúl 62% and Orlando 75%. The facilitators, especially undergraduate pre-service teachers (UGs), varied every semester, thus affecting whom students wanted to work with. Students usually remained in the same group but with different facilitators over time.

2. **Trajectories of Positioning and Participation Over Time**

In this section, I present an analysis over time of the different positions that were co-constructed during social interactions in small groups solving mathematical tasks. The analysis over time was developed through time series related to the chronological order of the data, which are the beginning, middle, and end of the program. Each segment is approximately one semester and a half of analyzed data. During this time, students attended 3rd through 5th grades. Although positions were negotiated in moment-to-moment interactions by the interplay of three factors—attention, alignment and ability—the analysis over time presents recurrent patterns of positions for all students. Yet exceptions emerged, proving the situated, flexible nature of the positioning process.

Four types of positions were identified. This identification process included the analysis of the quality of students’ social and mathematical participation while interacting in each group, noting how the process allowed students to have an active role during the meaning making process, access
resources, negotiate participation, receive support or attention from others, express interest in participating, and have equitable chances as other group members. Below I define each of these four kinds of positions, describing how the situation also affects the participation of the students and also their opportunities to receive attention, alignment from the rest, and support for their actions and performance during a problem-solving task.

a. **Powerful Mathematical Positions**

The student in this position has central participation reflected through social and mathematical attention from the group. Peers and facilitators are likely to align with the actions of this/these student/s. Often the person with a powerful position discusses and collaborates with others in doing what s/he needs or wants to do. Even when s/he does not understand the task, s/he receives immediate attention and access to resources to guarantee ease in the process supported through social actions, such as being listened to and having her/his ideas taken up and elaborated on, at times being asked by others to explain or do the task, and being praised for the explanation. This situation may also promote a dominant approach, in which students with powerful positions may not necessarily share decisions, resources, or needs with other members in the group. Several powerful positions may emerge in a group, but they might not support each other.

b. **Equalized Mathematical Positions**

These positions correspond to times when the attention and opportunities are distributed among participants, so that collective action and goals evolve and are supported. It is a multidirectional process since various students can have access to the task, be heard, and be attended to. Decisions, problems, approaches, and resources are negotiated and clarified with each other. Thus, creative and helpful solutions emerge collectively in joint productive activity. Ability is distributed in the group.
c. **Powerless Mathematical Positions**

The student has peripheral, marginal, or secondary social and/or mathematical participation. A student in this position can receive attention that does not align with her/his needs or interests. Thus s/he can be publically singled out without any benefit besides being marked as different or, at times, silenced. This may also escalate into not receiving any attention whatsoever. Students in these positions have little to no support from the group on their decisions. These positions lead to isolated performance or participation, but they may also emerge in discussions or collaborative interactions that flow in one direction only, which is not that of the student with the powerless position. The process may lead to relinquishing the goal or task at hand.

d. **Null Mathematical Positions**

These represent the type of student participation that is not in the center of the interaction, but not entirely ignored either. This represents a mathematical position in which the student may accomplish the task, but it is never certain whether s/he really understands it. This student has an alternative series of participation that are not affected by social interactions. The level of attention and alignment is weak, but not too scarce; they promote performance that is just enough not to fail. There is no interest, portrayed through the student’s or others’ actions or comments, in collaborating during mathematics, but if the wish is expressed, it is taken up. Here the student may develop peripheral to central mathematical, social participation or performance but without a clear attempt to do so, neither from the student him/herself, nor from others.

Now I present the different recorded positions that the selected students accumulated over time in their participation in the afterschool. A total of 32 sessions per group/student were analyzed, recording the different positions that each student case had during each of the afterschool time segments. Each segment (i.e., beginning, middle, and end) or graph below represents 8 to 11
recorded sessions. The positions were converted into percentages and displayed in Figure 10 (see below) in which the percentages of the different positions that students had are color-coded (blue = powerful, red = powerless, green = equalized, and purple = null).

The results above portray how Raúl had stable powerful mathematics positions over time. Betty and Orlando experienced diverse positions throughout, yet they had high percentages of powerless
positions in the beginning (45%) and in the middle (50%). These percentages decreased at the end of the program (Betty at 33% and Orlando at 30%). Percentages of their combined equalized and powerful positions were higher at the end for both of them. I present here possible explanations for these positioning trends for both of these students. On the one hand, it could relate to their becoming familiar over time with the structure of the program. However, this pattern does not seem to hold for the other participants. Indeed, the increase in powerless positions in the middle of the program indicates that it could relate to some other factor. On the other hand, changes of facilitators could be a possibility for Orlando who had different facilitators between the middle and the end of the program. Yet Betty had the same facilitators during these two times. During the beginning and middle of the program, Betty had different facilitators and Orlando had the same facilitators. Therefore, the increase of powerless positions for these students was not directly linked to which facilitators they had. Peers do not seem to be related to these possible changes, as some were present throughout the program in the same group with the student cases. The most common relation across the analysis here is the presence of the two boys or two girls in the same group.

A possible factor for these changes may be a loose correlation to the curriculum or what tasks students worked on during each of the three segments of the program. When exploring patterns across these units, it becomes clear that the ones included in both the beginning and the end of the program had tasks that were adapted specifically for the students in the program and based on their interests. Thus, tasks included here promoted investigations and experiments through game-like situations or challenges. Students also created their own problems. In contrast, the units included in the middle of the program were different from the other two segments in that the tasks were not adapted to the students but merely translated into Spanish. This segment also included a digital story project. Both projects developed in beginning and middle promoted mathematization
processes, but the earlier one had students explore the games they had created. In the later one, students mathematized the practices of community members and not their own practices. Although both were open-ended projects, the former aligned more closely to students’ practices and knowledge (Bereiteer & Scandamalia, 2003; Freudenthal, 1991; Gravemeijer, Cobb, Bowers, & Whitenack, 2000; Garrison and Mora, 1999; Lo Cicero, et al., 1999; Streefland, 1991). Thus, I believe that the context promoted by the units in the beginning and the end of the program seemed more aligned to students’ interests and supported their collaboration and ability better than units in the middle segment. In the middle segment, even students with high social status had fewer powerful positions.

Interestingly, Fabiola and Betty had nearly inverse positioning patterns during the beginning of the program in comparison to the middle and end. I explored this particular time, as it could open possibilities to better understand positioning at this time. All student cases worked in different groups the first months. At this time, Betty had higher levels of powerful positions and Fabiola powerless positions, but both boys had similar patterns as they did later. Intrigued by these different positions, I analyzed more thoroughly the facilitators’ fieldnotes from this period. I explored how facilitators’ narratives portrayed students’ actions. I tagged and categorized all the instances when any of these students was mentioned.

In Figure 11, I present the frequency of student descriptors. Facilitators’ narratives portrayed the students as follows: a) Orlando as someone with low confidence in mathematics, with high social interest, and who used mostly Spanish; b) Raúl as someone with great mathematical confidence, and high participation and who used both languages equally; c) Betty as someone with great social and mathematical interest and participation and who used more English than Spanish;
and d) Fabiola as someone with interest in mathematics but with low related proficiency, who was not seen as competent, and who used English only.

Facilitators’ perspectives on Betty and Fabiola are nearly the opposite of the results of the analysis over time. Yet these contrasting perspectives portray the flexible and situated nature of positioning.

The different groups that the girls worked in might have promoted different perspectives and positions for both of them. Some excerpts from the facilitators’ fieldnotes remark that:

a. Jacqueline [another student] was quick to figure out that she was going to pay me 2,000 but Fabiola didn’t have a clue as to how or what I was going to charge next. I worked with Fabiola more as Jacqueline was running around collecting more things to add to her collection. It took Fabiola a while to figure out the answer.

b. After Jacqueline had finished I asked her if she could try to explain to Fabiola how she figured out the answers. When Jacqueline tried to explain Fabiola still looked confused, I saw Fabiola was just copying down whatever numbers Jacqueline had on her paper and move on.

These excerpts portray Fabiola as someone who did not understand the tasks well and had difficulty addressing the mathematics. Fabiola was also slower to process the tasks and needed more explanation or attention from both Jacqueline and the facilitators, and still she did not improve.
When I explored some of the interactions in the videos, it seemed as if Jacqueline did not have the patience to align her actions and interests with Fabiola’s pace and ideas. Jacqueline could not slow down and pay careful attention to Fabiola. Jacqueline was more interested in the facilitators than her peers. Both Fabiola and Jacqueline spoke in English only. Fabiola’s powerless positions, then, seemed related to Jacqueline’s levels of social and mathematical dominance and to the infrequent alignment to Fabiola’s actions and interests. These situations placed Fabiola in peripheral participation, often with powerless positions. Early narratives of the facilitators about Betty also portray a different perspective about her mathematical fluency and understanding of the problems:

a. Betty took the lead and read them both in English and in Spanish. I asked them [the group] if they had understood the problem and they said no but Betty said maybe and […] I asked why and she said it said so in the beginning; which meant that she read key words to understand the problem.

b. Betty noticed right away that each quarter has a value of $0.25 and that there was 12 quarters all together. Then she divided the quarters so that at the end there were 4 groups…

c. Usually when she explains a math solution she says it in English. […] she was very concentrated. The carnicero problem… which she knew very well. Betty started counting off 5,10,15,20 and so on. She did her problem another way and counted off by 3’s, 3,9,12, and so on. That amazed me because out of the three kids, she is the only one who knows multiplication and division well.

These excerpts portray Betty as someone bilingual who knew mathematics, participated more than others, and understood the tasks right away. One aspect became prominent by contrasting Fabiola and Betty’s general patterns of positions during the first semester and the rest of the program, that is, the group configuration. Coincidentally, some facilitators commented on it as well:

a. Last week I noticed that Betty tended to roll her eyes a lot and had a bit of an attitude when she spoke to Jenny or the other kids in her group but today in this new group I saw her raising her hand and although she wanted to be first in dominoes I didn’t see her rolling her eyes as much or talking to the girls or the facilitator with an attitude. Fabiola switched groups because she didn’t feel comfortable working with Jacqueline. Now, Fabiola seems to have a closer relationship in the group.
It is as if both Fabiola and Betty did not seem to get along with their previous peers as much as they did with the newer group. For Betty, the issue seemed related to finding the first group boring, more childish, and using more Spanish. The newer group peers seemed more mature to her. They resembled teenagers and spoke more English. In this new group Betty was considered more Spanish dominant than other peers. For Fabiola, the first group spoke English only, and she often found tasks difficult. Her peer was self-centered and over-focused on adult attention. In the newer group, Fabiola spoke both languages and became the leader, determining the group’s pace. The boys’ group was more stable, due perhaps to the fewer number of male participants. Regardless of student choices to form groups, the process seemed connected to choosing partners based on personal interests that mediated the permanence of these networks that also affected their positions.

Nevertheless, the total of positions across four student cases and the different segments of the afterschool show specific trends in how positions evolved over time (See Figure 12). Powerful positions had the highest frequencies of all types of positions. Powerless positions were the second most frequent, but they decreased over time. Equalized positions instead increased their frequency over time. This inverse frequency distribution between powerless and equalized positions portrays a progression of group interactions around mathematics aligning closer to collaborative approaches.

**Figure 12: General positions over time by type**

![Diagram showing frequency of positions over time](image)

Figure 12 shows that in middle of the program there were fewer powerful positions and greater null positions for all students. These totals seem to parallel general percentages of language use. In the
beginning Spanish was used 37% of the time and English 63%; in the middle 22% and 78%, respectively, and in the end of the program 31% and 69%, respectively. These results are not intended to claim that higher percentages of using English decreased the opportunities to access powerful positions. However, this pattern resembles Fabiola’s situation during the first semester when she used English only and had more powerless than powerful positions as well as her situation when she later used both languages and her opportunities for powerful positions increased. Perhaps when Spanish was placed under ideological power dynamics, those students who were linked to it, like Orlando and Betty, were more likely to face powerless positions, but when Spanish was discursively used in mathematics, this situation may have afforded greater chances for equalized and powerful positions for bilingual students as they worked together in mathematics, accessing their linguistic resources as needed.

E. Summary of Positioning Over Time

Students’ powerless, powerful, null, and equalized positions accumulated over time seemed related to their previous high or low social status and school grades. Yet the positioning process is flexible and situated since social interactions may afford any type of position to any team member. Still, the process of identifying oneself as a doer of mathematics seems connected to recurrent positions. Powerless positions evolved through misunderstandings or poor quality of attention, often related to narrow perspectives about accepting and elaborating on students’ unconventional discursive practices. These ways were either marked as wrong or ignored, and more conventional perspectives were legitimized instead. These situations depict biased social processes empowering certain forms of being and doing closely connected to a culture of power that legitimizes who is a doer of and what counts as mathematics. The process of positioning also seems related to the quality of the tasks and the kind of groups in which students participated. These variables seemed to affect
how interactions evolved and what students focused on, thus also promoting different forms of positioning for students. However, since the positioning process is flexible, it can be neutralized or manipulated. There were times when equalized positions emerged. The general positioning patterns showed that the number of equalized positions increased over time across groups. This could reflect how students may start developing more collaborative and distributed actions as projects and work in groups are promoted. The quality of equalized interactions recorded here may inform these attempts to promote collaborative work.
VI. DISCUSSION AND IMPLICATIONS

A. Introduction

In the previous chapters, I described the relevance of the social dimension during students’ participation in mathematics, specifically how the quality of participants’ interactions affects their positions during problem solving. These positions were co-constructed in the situated ecology of interactions with others and affected by different elements (e.g., tasks, groups, mathematical practices, and language) that contributed to students’ opportunities to access resources, reasoning, and quality of social relations while working in their groups. As a result, students accessed equalized, powerful, powerless, and/or null positions in mathematical or social areas. Often the positioning process was influenced by lines of power which promoted assumptions about students’ actions and competence. These lines of power seemed related to the recurrent positions of certain students over time.

In this chapter, I will discuss this positioning process and its implications and significance for teaching and research. First, I present a summary of how I understand the positioning process as well as how the factors identified in this phenomenon work together. Then, I discuss positioning over time and its relation to lines of power manifested through language and students’ perceptions as bilingual doers of mathematics. Next, I discuss the implications of the phenomenon for teaching and learning. Finally, I present limitations of the study and suggest areas for future research.

B. Summary of Positioning

Positioning theory and the participation framework have described that positioning and the quality of participation are situated processes that are co-constructed by participants who assign and take roles through discursive actions. With these ideas in mind and the dialectical connection between learning and participation in human activity, this study explored how all these elements
interact during the positioning process during group work solving mathematics problems. When the group of students met to work on a problem, all participants in one way or another were already, even without speaking, communicating their intention of participating in that group by both coming to the afterschool and sitting at the table with that particular group. As they started gazing at each other or talking with other members, the process of communication became more obvious as their responses elaborated on what was occurring or not. This promoted different processes (attention, alignment, and ability) which seemed relevant the quality of actions (e.g., talking, reasoning, understanding, clarifying, hypothesizing, checking solutions) that were promoted for all students as they problem solved with others.

But how do these processes mediate positioning and participation? The collaboration process was constrained by the context of the problem-solving activity at Los Rayos, which asked students to meet in groups, pick a task they wanted, and solve it together. This situation already created expectations to work and interact with each other. This initiation phase was crucial in the positioning process because this initial situation already hinted at the participants’ understandings, interests, and competence in the task, which also contributed to the development of assumptions about others which could be either elaborated on or shifted. Various authors such as Rogoff (1990) and Lave & Wenger (1991) have discussed how silent or peripheral participants can actually learn through active observation, a kind of indirect but guided participation. They argue that these ways of participation are legitimate ways of learning. However, Rogoff (1990) contrasts two participation structures: first, the intent participation tradition, where experts play a guiding role, facilitating involvement alongside learners, both in fluid interchange; and, second, the assembly-line instruction in which experts manage learners’ communication and behavior. For the latter, she argues that the “learning of some is fostered while the rejection of others as learners is also sought, in line with
bureaucratic needs for efficiently sorting individuals and life opportunities” (Rogoff, 2003, p. 196). As I disagree with the latter and recognize the former stance on active learning, in my exploration of positioning, I see both stances mixing through the negotiation of participants’ opportunities and roles to engage in problem solving. I use the diagram below (Figure 13) to describe how I envision the interplay of the three positioning elements or factors (i.e., alignment, attention, and ability).

**Figure 13**: Factors mediating the positioning process in mathematical activity

I use the triple “A” model in Figure 13 to describe how I understand positioning as a type of affordance or product of interactions that both happen to participants and that they have access to during their interaction with others. I see the positioning process co-constructed by the self (me) and others at the interplay of three factors (alignment, attention, and ability) that emerge during and also
shape the interactions in mathematics problem-solving activities. These factors also can be influenced by lines of power affecting how students’ actions are evaluated or perceived by others under the influence of ideological perspectives and assumptions about language (Spanish and English), a type of discourse (formal and informal; conventional and unconventional), and collective or individual processes. Therefore, based on the analysis that I developed, I envision social interactions as processes that can either support one’s and others’ performance in problem solving, or, contrarily, they may limit these circumstances depending on the kind of contradictions that emerge as people come together. In either case, the positioning process of participants is activated. With this comparison I do not imply that the process can evolve only correctly or incorrectly, but that it is a complex process in which different elements interact.

As part of describing the complexity of the process of positioning, I identified three main factors that evolve during social interactions and mediate the positioning process of the participants. As participants pay attention to, align with, and assume ability about each other, they develop their interaction, and in the process, they also position each other. I will describe each of these factors and how they contribute to the positioning process. First, alignment refers to how participants coordinate their actions with those of others during an activity. Alignment is manifested by what participants do or say to each other and how they respond and elaborate on each other’s actions around a task. If a person coordinates her/his actions with others and vice versa, then this person would be more likely have a strong position in the group, such as a leader. If that person’s actions also target the control of and resources in the group and the rest of the group accepts and supports this process, then that person has a powerful, dominant position.

The second factor, attention, refers to a process of noticing or focusing on one or multiple aspects in the environment of interactions (e.g., the task or a participant’s actions or needs). This
focus of attention could be shared among participants or not. As the focus of attention emerges and some aspects are noticed, others are ignored. This selection determines what matters to the person or the group at that moment. What matters to one or others can create bridges or barriers between people and consequently affect positioning. If a group’s focus of attention on a problem-solving task is divergent from an individual’s, this individual is more likely to position her/himself or be positioned by others as different, and disagreement may emerge.

Although the convergence of the foci of attention may seem like alignment, I differentiate these processes by centering the definition of alignment on the process or attempt to coordinate one’s actions with others. Instead, attention refers to the process of focusing on something, or what matters to the person or group at that moment. Therefore it could be possible to have alignment with different foci of attention. Imagine a group working together in which the members are coordinating or aligning their actions. They do not necessarily need to agree in every dimension, given that they may pay attention or notice some aspects differently. Participants in coordinated social actions may notice different elements regarding the task, others, or oneself. This situation promotes the risk of developing different foci of attention that may transfer to positions that could still be re-negotiated through attention and alignment processes from all participants. Consequently, it is also possible to have shared focus of attention but not alignment among participants. This situation could evolve by the lack of alignment (disalignment) of a member and a group where both parties do not coordinate their actions, but still have their focus of attention on other participants. For example, in a group a student was punished or excluded from the group’s interactions for not having followed the rules, but even when he was excluded, the group paid attention to what he was doing. So, in this circumstance what the student did still mattered to the group, but there was no clear attempt from
either party to work together. Thus alignment and attention inform each other, and when they intersect, they promote participants’ equitable negotiations and opportunities.

The third factor, *ability*, refers to different ways in which participants are perceived or co-constructed as competent—or not—during problem solving. Although ability relates to a person, his/her ability is constructed through a social process that supports his/her performance in an activity based on how others assessed that performance. The ability of an individual evolves when the noticed performance—or lack thereof—is validated by others through a process of judging, comparing it to others’, and deeming someone’s actions, verbally or nonverbally, as valuable, accurate, or pertinent to the task at hand or in relation to what others do and say. At times, students with unconventional, but potential ways to solve a task productively were not supported by others, as their performance was either not understood or deemed as wrong. This situation often limited the meaning making process and lowered these students’ performance or competence during problem solving. In such circumstances these students faced powerless situations as they expressed their own ways of thinking. Since they were perceived as wrong, these students had unproductive or no attention from others and also little attempt or action coordination to understand and/or help them. Ability, then, goes beyond traditional definitions that situate it in an individual’s aptitude. In this study, ability was also connected to language ideologies since some students, who either had difficulties using English or used unconventional discourse, were less supported and also considered by other peers and facilitators as Spanish-dominant speakers and thus less competent students.

Therefore, it seems that alignment, attention, and ability in connection to the lines of power (collaboration, mathematical practices, and bilingualism), mediated through discourse, affected the positions and participation of the members in same group. I identified four mathematical or social positions that were constructed during interactions: a) powerful, b) equalized, c) powerless, and d)
null positions. I analyzed these positions that developed during social interactions, thinking of them as a social activity of mathematics problem solving that had the object of actively engaging students in mathematical activity with the purpose (outcome) of learning or developing understandings of new mathematical concepts through the use of resources such as language, manipulatives, beliefs, previous knowledge, interactions with others, and the tasks. I used the idea of contradictions or tensions among components (Engeström, 2001) to explore what elements of the social interactions in the activity of problem solving could have promoted these contradictions. I noticed that positions were linked to conflicts or shortcomings developed in any or several of the elements in the interaction, thus affecting the way alignment, attention, and ability were constructed in the groups. In powerful positioning, others center their alignment, attention, and ability about the subject with powerful positions and away from others. In powerless positioning, the process was the inverse of powerful positioning, where others exacerbate the shortcomings or contradictions in their alignment, attention, and ability against the subject with powerless positions. In null positions, the subject and others share and give low quality attention between alignment and ability in the activity. Finally, in equalized positioning, all participants continuously activate in all directions their alignment, attention, and ability in the negotiation of contradictions or tensions that evolve.

Furthermore, other aspects seemed to affect positioning in how alignment, attention, and ability evolved in the different groups. These aspects seemed related to lines of power and the context of the social interactions. These aspects are 1) Tasks, 2) Discourse and Codeswitching, and 3) Grouping. In the following, I will address each of these.

1. **Tasks Affecting Positioning**

   The relevance of tasks in the positioning process seemed connected to two levels: constraints regarding either their facilitation or their resolution. During facilitation, the development
of positions seemed independent of the facilitators as personal individuals, but their role in the process was crucial (Delpit, 1988; Nieto, 1994; Nunes, 1999; Mercado, 2001). Their roles seemed limited by the task structure, which mediated how they interacted and were willing to adapt to students’ reasoning and difficulties. When the structure of the tasks demanded more linear processes and/or solutions, facilitators seemed more open to negotiating or paying attention to students who had difficulties or divergent behaviors during problem solving. Perhaps the predictability of the solution allowed them to expand their attention and alignment to students’ concerns, needs, and divergent ideas. At times, this resulted in re-adapting the task, so that students could successfully reason about and solve it. Conversely, when the tasks promoted more open-ended trajectories, the facilitators’ attention and alignment seemed dispersed between the mathematical concepts and the students’ actions. This situation apparently led facilitators to focus on and support the actions and ideas of students who seemed to understand the task better. Consequently, students with divergent ideas were less likely to catch on and had powerless positions. Nevertheless, in this same type of task, when there was a strong collective process and students interjected freely to resolve differences, the students had equalized positions. Therefore, as current mathematics approaches support the benefit of open-ended tasks with high cognitive demand, which give students more chances to explore mathematical concepts, it is crucial for teachers to plan how to support their thinking and collaboration as they work on those tasks. Given the circumstances that evolve in different groups, facilitators need to pay more attention to all students in order to provide successful scaffolding and be aware of not privileging those students who seem to know the most and not limiting the opportunities of students who struggle in these tasks and might need the most support. As Van Lier (2004) argues, “scaffolding develops between the planned and the unpredictable and higher skills emerge in untried actions, but both demand from
students and facilitators careful attention of when to elaborate, so the new emerges from the known but from learner’s initiative” (p. 162).

Regarding the resolution of a task, the analysis over time of students’ positions showed that all students had higher percentages of powerless positions in the middle of the program than in the other segments. The explanation that seemed most likely to explain these differences was the kind of tasks included during this time. When comparing their quality in relation to the task in other segments the trend was that the tasks in the middle of the program seemed to have less direct connection to the students’ interests, experiences, and knowledge. Thus, this indicates that from the very beginning these tasks presented a gap that was difficult to bridge. With this I do not mean that students should not be challenged, on the contrary, but in order to become productive, tasks need to connect with what students currently know and then expand on it (Van de Walle, 2004). Therefore, it is necessary for teachers and facilitators to explore how the tasks represent groupworthy tasks (Cohen, 1994) as well as the task’s levels of cognitive demand (Stein, Smith, Henningsen, & Silver, 2000) and authenticity (Newmann, Secada, & Wehlage, 1995) so that students would be more likely to be engaged, interested, and successful in the tasks.

2. **Language Affecting Positioning**

Students’ quality of discourse played an important role in how others, especially facilitators, noticed and aligned with them. Mathematical discourse practices refer to different ways of doing, thinking, talking and believing about mathematics (Moschkovich, 2003). Yet discourse is a contested space of power construction, inherently ideological, and it defines what is acceptable and marginalizes values and viewpoints (Fairclough, 1989; Gee, 1990; Planas, 2005). On the one hand, when students did not understand or expressed their ideas in unconventional forms, assumptions about their low ability evolved, giving them powerless positions. On the other hand,
when students articulated their ideas with discourses assumed to be clearer or more mathematical, facilitators validated their actions and assumed students’ higher ability, thus giving them more attention and alignment and putting them in powerful positions. Peers reacted similarly, but their influence was not as great as that of the facilitators. Results from encounters like these simply reified positions. Those who earlier knew more also learned more through a supported, scaffolded interactive process in problem solving. Therefore those needing the most attention actually had the least. I believe that this could be linked to the overemphasis on the object of the activity which, in turn, overshadows the main goal of the activity, which is that the students, all of them, achieve the object and eventually the outcome of the activity (i.e., learning more and creating stronger relations with mathematics). Although the goal is enhancing mathematical activity and ability, they are both still socially mediated.

Another aspect of language was codeswitching, which was used continuously as students worked bilingually across tasks. During the interactions of positioning events, codeswitching functioned in different ways (Cashman, 2008, MacGregor-Mendoza, 1998, 2000; Trujillo, 2005). For example, at times it was used to mark a difference among students. The process of showing differences was used in order to correct errors, to clarify ideas, to other or mark someone as different from the rest, and to patronize. These processes emerged when, within sentences or conversations in a given language, participants briefly codeswitched to another language. Interestingly, patronizing comments occurred most often when there was a mistake or a mathematical misunderstanding. At the same time, Spanish was the only language used for this purpose. This seemed to occur for two reasons. First, students used English more often and switched language to mark differences. Second, as shifts were related to mistakes, the use of Spanish made errors obvious by using the language that the struggling person understood.
Switching languages was thus connected to language power differential. The second use was connected to this line of power but by making use of English during mathematical discussions to gain authority. The analysis of language frequency demonstrated that, although students used both languages during overt discussions in groups to decide what language to use, Spanish was often subordinated and English deemed as the choice or the legitimized language to use. This was the case even in with bilingual Latinas/os students working in a bilingual setting with bilingual facilitators. Finally, a decreasing use of Spanish during public self-speech was also observed. Spanish use was gradually subordinated during self-speech (Vygotsky, 1978) in public social spaces, especially if the discussion was in English and despite the fact that all participants were bilingual. Students made use of self-speech in Spanish only when others did not notice. Although positioning was not directly negotiated here, their use of Spanish had a powerless position. These facts placed the linguistic resources that students had learned at school in a powerful position and excluded or put in a powerless position the linguistic resources students had learned at home. When language ideas were contested in light of power, either language worked as a sorting mechanism related to clear positions. As Fairclough (1989) argues, “certain types of discourse acquire cultural salience and ‘colonize’ new institutions and domains” (p. 73).

3. **Grouping Affecting Positioning**

Two of the students in this study (Betty and Fabiola) experienced different, almost opposite, positions between the first group where they participated briefly and the group where they remained for the rest of the program. These patterns seemed to describe a possible connection between who is in a group and how social interactions and positioning may evolve. However, the positioning trends for both boys seemed recurrent regardless of the group where they worked. These situations could relate to concepts like Expectations Theory (Cohen, 1984), assigned identities
(Sfard & Prusak, 2005), beliefs about self-efficacy (Bandura, 1997), or adaptive adequacy (Steele, 1988). But regardless of the groups where students interacted, they all experienced different positions. Positioning shifts portray its fluidity and situated nature. Therefore, these situations support the possibility that a careful process of attention and alignment (van Lier, 2004) will support higher levels of student competence and cease the practice of having fixed expectations for students.

C. **Positioning Patterns Over Time and Students’ Perspectives**

Despite the flexible and situated nature of the emergence of positioning, analysis over time of students’ positions showed a recurrent pattern that resembled previous academic performance. However, the analysis presented in this study portrays only one way of understanding the process of how students’ interactions affect their positioning. In two of the examples presented in this study in the findings chapter, the process of positioning was evident to students also. Orlando complained to a facilitator who had excluded him from the mathematical task, since the facilitator had focused on Raúl only. Similarly, while having difficulty solving a task, Betty described to her facilitators that she was self-conscious about her mathematical competence because she often found mathematics difficult. For both reasons, I found it relevant to explore what these students said during the interviews about positioning and how they interpret their experiences in mathematics.

Students with previous high social status and grades reported seeing themselves as having expanded their understanding of mathematics through their experiences in the afterschool, not only regarding quantity, but also quality, as they argued: “We saw the fun side of the math.” Likewise, students with previous low social status and grades cited having enjoyed the activities in the afterschool, which were mathematical in nature. They, however, did not mention having improved their perception of themselves as doers of mathematics. Instead they reported: “I’m not smart,
they (points at a student with high social status and grades), *they have a big brain and always do it all*” and “*I hate math.*”

These students’ perspectives parallel this study’s results. By no means do I claim that the approach utilized here can describe students’ experiences as they actually interpret them, but my take on this is that the process of positioning, the immediate interactions among people and how they are positioned and how they position themselves in an activity, may in some way also affect how they think of themselves regarding that activity and their competence in the practices which that activity encompasses. Positioning, in its recurrent patterns, could be related to and inform how students see themselves in that area. There is no evidence from the class and other settings besides the teacher’s informal comments and students’ grades, but they all seemed to point out the same pattern, namely that students who accumulated greater percentages of powerful positions (Fabiola and Raúl) claimed to be English dominant, to like mathematics, to have learned more, and to have explored the fun side of mathematics through the activities in the program. Conversely, students who accumulated greater percentages of powerless positions (Orlando and Betty) self-identified as balanced bilinguals and perceived themselves as incompetent in and disliking mathematics, but still they reported having enjoyed the activities in the program. Simultaneously, these students compared their ability and opportunities to the positioning process that they had seen privileging other students (Fabiola and Raúl), those who seemed to be smarter and always received more attention from the facilitators and more chances to do complex mathematical tasks. One could deduce that students are able to see the different positions co-constructed during the social interactions in which they take part. Similarly, the accumulated information about one’s and others’ positions may also provide an accumulated perspective, like a bank of information about oneself and others, that is, a socialization process.
Ochs & Schieffelin (2008) argue that the language used around people socializes them into certain identities and discourses about themselves. In this study the language in social interactions around the mathematics and bilingualism performance would socialize students in certain ways of seeing themselves. Lantolf (2000) writes that “the motives and goals of particular activities cannot be determined solely from the level of concrete doing, since the same observable activity can be linked to different goals and motives and different concrete activities can be linked to the same motives and goals” (p. 8). Perhaps, then, the conditions, affordances, and positions that students experienced during interactions in mathematics problem solving helped them realize the goals and motives of their actions in this context. In this way, although all students discursively participated in tasks called “mathematical” by the afterschool program, only some of them (students with high social status and grades) were able to understand their activity as mathematical, since they had the affordances to view them in this way. Likewise, other students (those with low social status and grades) realized that their actions in those same tasks were fun activities at Los Rayos where some students did mathematics (students with high social status and grades), but not they themselves because their self-perspectives or subjectivities, mediated through their recurrent powerless positions, informed them that mathematics was not for them. Although they successfully completed mathematical tasks, they did not perceive them as mathematical. For example, Orlando mentioned that he liked the mathematics “we did in Los Rayos,” and he also said that “I’m not smart, but they have a big brain and do it all.” Yet when he successfully completed a task, he often cheered up and seemed interested in having others notice as well. However, in these situations, he often had the assistance of another facilitator. It is uncertain whether he would identify ability as that of others, of himself, or both. On the contrary, Raúl, who often received support from facilitators, understood the mathematical ability clearly as his own. In his case, the situation was usually supported during
interactions but not overtly mentioned. However, the perception of his ability came up during his own and others’ interviews. Thus, their narratives seemed to reflect their perceptions of the roles, positions, or goals that they took on, were socialized into, and realized during their discursive activity and not directly from what they did.

Regarding language, only students with low social status and grades saw themselves as bilingual speakers, but not as doers of mathematics. However, just as all participants in the afterschool, they used English most of the time. Students with high social status and grades saw themselves as doers of mathematics and as English-dominant speakers, but not as bilingual. However, just as the others, they also made continuous use of both languages. Again, their narrative and discursive actions seemed contradictory. In order to try to understand these situations, I juxtaposed them with interactions where lines of power seemed evident. First, overt negotiations of language and students’ use of self-speech subordinated Spanish. The greater power of English became obvious during negotiations regarding the question: “What language do you want to speak in?” and “Are you bilingual?” These negotiations seemed to activate larger sociopolitical ideologies about being American. As students faced these questions, they also faced a stereotype threat. Previous studies on stereotypes have found that, when facing a situation of being stigmatized through a stereotype, people or groups disidentify themselves from the stigmatizing domain by compensating with another as an attempt to rescue their self-esteem (Goffman, 1963; Murphy, Steele, & Gross, 2007; Steele, 1997). In fact, Cheryan & Monin (2005) have provided information on how Asian-Americans, often seen as less American, and under similar circumstances, overemphasize being part of practices perceived as especially American (i.e., playing football). Therefore, when students were publicly confronted with these sort of questions, the process of publicly accepting a strong Spanish-speaking identity would confirm the threat of having a separate
history from what they feel a part of and want to belong to, namely a legitimate American identity, which, in the current sociopolitical context, seems divorced from having a bilingual identity. We therefore need models of bilingual Americans (Brisk, Burgos, and Harmela, 2004).

Secondly, students with low social status and grades were seen as less fluent in English although they used it most of the time. It seems as if their “errors,” when they had little support from others, were often linked by their peers to their “lack” of skills in the language, and they were positioned as non-fluent English speakers. Perhaps their low English status prevented and excluded them from identifying with a dominant language, and instead they did so as bilingual speakers. Although being bilingual might be considered an asset, I wonder if they see it that way. Just as Jimenez (2002) states: “Qué bueno que las indias no hayan ido a las escuelas de caciques, porque en su marginación conservaron el aprender de la vida y la comunidad. Good for female Indians that they did not attend the chief’s schools because in their marginalization they preserved the learning of the life and the community.” Marginalization in a way validates isolated diversity, but the challenge is to capitalize on diversity in the same context and at the same time.

Although these previous relations could seem to render the possibility that recurrent patterns of mathematical positions over time may influence how students see themselves as bilingual doers of mathematics, two questions emerge. First, is the positioning process the result of already established patterns that simply manifest themselves through interactions? And second, how are these positioning patterns developed differently in the afterschool from those in their regular classroom and across classrooms (and teachers), at home, and with friends? Cohen (2000) shows that social status carries over from setting to setting, that it is obvious to oneself and others, and that it affects how one interacts with others. Nevertheless, it is not fixed and can be modified, at least momentarily during immediate interactions, through explicit and direct interventions of assigning
competence, with the goal that students start seeing themselves in a different way, too. Although I consider the pursuit of these questions very interesting and I can also see the relevance of Cohen’s argument, I still think that the relevance of this study is not in demonstrating that social interaction affects positioning and that positioning is a flexible, yet recurrent pattern. Rather, I think the relevance of this study is that it identifies some of the processes through which it occurs. Further research can also explore these processes in different settings to learn more. The main goal is to bring us to reflect on how we notice students, how we align with them, and how we perceive their ability based on the assumptions we make in relation to using language and the tasks that we select and engage in. Similarly, this could be a process that students also become aware of, so we all can begin the process of distributing ability, negotiating difference, and developing equalized positions.

Therefore, it seems that it is through this positioning process that students form their ideas about contexts in which their actions are legitimized and thus where they belong or who can be a bilingual doer of mathematics. Shields, Bishop, and Mazawi (2005) argue that colonizing discourses are maintained and perpetuated through power relations that create and mark differences, positions, and pathology. Gumperz (1986) claims that positions related to ability lead individuals in lower positions to view themselves as failures. Thus, “the production of failure is as much part of routine collective activity as the production of average, ordinary knowledgability” (Lave 1988, p. 12). Martin (2006) asserts that students who are marginalized in mathematics need to overcome barriers by understanding the situations (social, historical, and political) that created their status as “minorities” and prevented them from becoming doers of mathematics. Cotton & Hardy (2004) state the need to disturb normalizing discourses in order to promote equity by capitalizing on diversity and undermining the “naturalness” of certain practices.
Correspondingly, these facts reinforce the necessity of the negotiation of these contradictions among the activity components, which were often mediated through normalizing processes of power that affect not only productive collective activity, but also the subjectivities and relations that individuals develop in turn. Nevertheless, these contradictions may be either invisible or undiscussible for the community where they evolve (Capper & Williams, 2004; Engeström, 2001). Whereas the invisible ones include normalization practices, the undiscussible ones are either covert or overt discriminatory practices. We need to bring these contradictions to the surface and negotiate them through stimulating a developmental dialogue (Murphy & Rodriguez-Manzanares, 2008).

D. Implications for Teachers, for Learning, and for Future Research

The process of positioning is both recurrent and flexible. This double nature of positioning makes it a hopeful but delicate process. Since it is not pre-established, it is flexible and situated in the local interactions that are co-constructed by the participants. Thus all participants have the possibility of having productive positions during interactions. Positioning is a delicate process because it is recurrent. This means that we are likely to develop similar patterns of interactions with a specific person as is that person with us, leading us to assume certain expectations about each other and to use them as guides of relating to each other. Moreover, accumulated patterns of positions over time seem to help students realize the goals of their actions in discursive activity. In other words, the recurrent positioning seems to affect the interpretation and meaning of one’s actions during mathematical activity, meaning which does not need to match exactly what discursively occurred, but which is supported by repetitive positioning processes. For example, although Orlando had sporadic episodes of successful engagement in mathematics, he mostly had recurrent patterns of powerless positions, and he recognized during interactions that he was not
getting the same attention as others: “Raúl always gets to do all things.” He also related this fact to what he perceived as very few opportunities supported by others to do mathematics and therefore being unsuccessful at it: “No, I can’t because they always make him do it.” He recognized that this pattern not only applies to him but also to others like him: “They always make him [Raúl] do it, not me or Armando or Juan.” In his recognition of his position, we can see how others’ perspectives contributed significantly to his “misbehaviors” that others used as occasions either to correct or ignore him, since he appeared unable to perform.

Therefore, because the positioning process distributes power among students differently, there are implications for the maintenance or dissolution of power differentials among students. Educators, especially teachers, need to recognize that differential positioning leads to differential power among students, and that power, in turn, results in positive self-concepts, different access to tools, and positive and supportive interactions which are the heart of learning. The positioning process evolves through the quality of participants’ interactions in three factors—alignment, attention, and ability—during mathematical activity. The quality of the social interactions around mathematics depends on how these factors are distributed and negotiated among the group participants. It is in this dynamic process, at the interplay of these factors, that positioning occurs and that equitable changes may begin. Still, lines of power may infiltrate these structures, creating disruptions and contradictions among and within the components of the activity. This is especially the case regarding beliefs about language and appropriate discourse in mathematics. In this complex process of positioning, collective actions are pivotal. Nevertheless, the facilitators’ actions often had a greater effect on how the rest of the group reacted to or positioned a particular student. Teachers must focus on enabling changes in participation, new forms of collective activity that are generated at the level of the classroom and beyond (Roth & Calabrese, 2004).
As teachers, therefore, it is crucial to explore the quality of our interactions with students. For this, the three factors of positioning and the presence of lines of power are important to consider. This process asks us to assess how we pay attention to students and how they notice each other as well. Is the attention distributed among students not only equally, but also equitably? Are there points at which some students notice the targeted concept or function and others do not? If so, how are these differences resolved? In this process of collaborating to solve differences, we need to consider alignment to coordinate our actions with those of the students, and, at the same time, we also need to encourage them to coordinate their actions with each other and the tasks. Are the tasks coordinated with students’ interests and knowledge? Do we align or coordinate our actions with the task’s goals or the students’ understanding of and needs in that task? Finally, since ability is the result of social support in a task and often based on assumptions, we need to explore what we understand as proficient and capable in mathematics in order to see what we currently value. We might need to explore further perspectives to support the diversity of students that we work with. Perhaps patterns in those who are succeeding—or not—in our classes may help us see what we are (not) supporting. Some facilitators could not understand the struggles or needs of some students until the students disclosed their perspectives about their struggles. The process of getting to know our students better is a way to start expanding our views about them and adapting accordingly. Students, like teachers, also need to widen their perspectives in order to understand others.

Likewise, as researchers analyzing and understanding teaching and learning practices, it is important to acknowledge the positioning process developed through interactions. Often students are portrayed as having difficulties without a deep exploration of the surrounding ecology created by social factors that may support students’ performance or not.
Current approaches in mathematics education place problem solving at the center of the processes of teaching and learning, and student collaboration in small groups is considered a crucial format of interactions supporting this goal (National Council of Teachers of Mathematics, 2000). Therefore, acknowledging the positioning processes as part of this goal is essential. Previous works have suggested processes to support collaboration by equalizing participants’ status and assigning competence to students with low status. Here, the teachers notice a student with low status and correspondingly reinforce her/his actions in front of the group, so that the student in question and the rest of the group notice this student’s competence in order to equalize her/his status and improve the quality of student collaboration. This process also includes thinking and developing groupworthy tasks that will genuinely bring the group to work together. Students are also convinced of different kinds of abilities and see that no student can have them all. This promotes accepting diversity within themselves and in the students they work with (Cohen, 2000; Cohen & Lotan, 1997). This approach, called Complex Instruction, focuses on the tasks and external actions of the teachers to promote changes. The results suggested in this study complement this perspective, so that we can think and work on what develops during interactions. Gumperz (1986) argues that school is not only about instruction, “but about how the information is made available, how ability is defined, accessed, evaluated; how knowledge takes place as it is socially defined and interactively constrained” (p. 68). When thinking about improving collaborations, then, not only teachers, but also students would benefit by knowing about how handling their alignment, attention, and ability in their groups can promote greater quality of interactions. This supports the process of beginning to listen to each other better, to accept and negotiate differences, and to place value on collective goals and actions.
A pervasive issue in positioning touched on the actions that peers and facilitators took based on assumptions about students’ discourse practices and language choice. Some students who used more Spanish than others and also some students who expressed their ideas in unconventional forms were more likely to receive little attention or to be ignored during interactions. As a result they were often misunderstood or corrected without an attempt to understand their points. Additionally, these students received “help” in that others switched to Spanish, but the quality of interactions often missed the needs of the student. The shifts were sometimes simply condescending with no clear intention of really trying to understand and help the student. Furthermore, some students started to subordinate their use of Spanish while doing mathematics, and others preferred to be recognized as English dominant speakers only. All these situations seemed linked to the interference of lines of power that place some practices (unconventional discourses and Spanish) at a low status. Technical language is a common source of power and authority (Lave, 1993) which needs to be carefully utilized. Discursive practices in school mathematics can sometimes be restrictive for students, depending on their social background, previous lived experiences, status in mathematics, and linguistic background. Instead, teachers and students need to recognize that it is not particular discourses that are relevant to a task but understanding and applying them meaningfully to their experiences, since it often disadvantages some students (Lerman & Zevenbergen, 2004). These perspectives need to be supported by teachers and the community of researchers, so that we can begin to better understand and support divergent actions or forms.

As a result, as teachers and researchers, we need to be aware of normalizing discourses and practices that promote these different perspectives affecting how we serve student diversity. In the context of this study, there were only bilingual Latina/o students and bilingual facilitators. Still, these normalizing practices were present and excluded those with less conventional actions. We
need a personal change that requires thinking of others as oneself, based on transformative intellectual knowledge (Sleeter, 2005), one that can help us challenge dominant discourses and positioning and overcome deficit thinking and discourses that pathologize the lived experiences of some children (Shields, et al., 2005). Thus, in our endeavor as educators, researchers, and administrators, we need to recognize these discourses and the positions they create as well as the ways in which they shape how we think, act, and decide, so that we can identify and move toward more inclusive perspectives.

E. Limitations of the Study and Suggestions for Future Research

The limitations of this study lie in its design (ethnographic case study) and context (afterschool setting), and the results emerged from and apply to these circumstances. Nonetheless, these features also represent the study’s strength, as it represents an extensive analysis of four students participating in two different groups during mathematics problem-solving tasks in a context that represents an unusual educational social space. The interactions in small groups focused mostly on mathematical meaning making, where students’ social interventions often shaped the tasks in various ways. It is precisely in this mixture of social and academic (content-specific) activity that this investigation is founded. I argue that this combination is relevant to the current mathematics field and the findings could inform but not directly apply to similar practices in other contexts and activities. This study’s participants were selected from an already existing group of participating students, so the data analyzed were already collected. Although the study’s original idea emerged through my own participation in the program, the specific analysis of the process of positioning evolved after the program’s completion and data collection. The findings, then, reflect a way of understanding how positioning emerged through an analysis that combined ethnographic experiences and data analysis. Results were triangulated through several types of data sources,
theoretical and analytical perspectives, and investigator triangulation. The analysis of students’ interactions and positions over time seemed parallel to and consistent with how students themselves described their experiences as well. The analysis of positioning was detailed and took extensive time; in real time, adapting to these differences would require keen concentration and observational skills. Thus, findings presented here specifically apply to this population. However, ideas that emerged here could be considered and adapted to explore similar issues in other contexts and then to corroborate results.

Much research needs to be developed across contexts and perhaps including more frequent interviews of the participants in order to have more information about their perspectives regarding their positions. It could be helpful to develop stimulated recalls (i.e., reflection on videotaped interactions in which one participated) that could illuminate their interpretations of the positioning events. Yet it is important to note that their interpretations might not always match their concrete actions, but rather the goals that they realized during interactions, not only in the original experiences, but also in the recall process. Since students with high and with low social status and grades seemed to have not only contrasting relations with mathematics through their narratives but also more fluid relations with mathematics in their discursive experiences, it would be interesting to explore immediate and distant reflections on their experiences in a regular classroom and across settings. In addition, I wonder what would happen if the context of the study promoted, reinforced, and/or challenged identity socialization as bilingual mathematical doers.

There is a lack of research focused on promoting difference, but especially how being different affects interactions in education between teachers and students on the one hand and between students on the other. There is a need for research that focuses on capitalizing on the perspectives of bilingual Americans and that investigates languages as invaluable resources, not
only linguistic resources, but also social ones relevant to learning and to being American. Ngo (2010) also calls for research that explores alternative American identities in order to decrease the othering process of the bilingual population that promotes unresolved identities:

While research explicating differences between immigrants and dominant US society has been important, the bicultural framework does not pay enough attention to the ways that the experiences of students as immigrants in US schools and society intersect with their experiences as adolescents, urban students and racial and ethnic minorities, among other identities (Ngo, 2010, p. 95).

Similarly, there is a need to explore further the nature and flexibility of mathematics learning application, not only how cultures or groups use and further mathematical thinking and learning, which is necessary, of course, but also how other ways of making sense and talking about mathematics are often othered based on normalizing discourses. I believe that the structure of the afterschool mediated students’ discursive engagement in mathematics and bilingualism, and as they engaged in both mathematics and bilingualism, they did not seem to notice that they were doing so. I believe that this was possible through the afterschool’s hybrid structure, which allowed for the mixing of various genres in relation to language and mathematical practices. Nevertheless, it seems as if the derived meanings were limited or unrecognizable. Perhaps these hybrid spaces call for new or more explicit forms of negotiation, representation, and positioning that may promote more noticeable ways of appropriating, validating, and realizing unconventional goals, so that students do not develop only temporary identities, but rather transform their self-perspectives as bilingual Latina/o doers of mathematics.
VII. REFERENCES


Dunleavy, T. (2011). What Makes Me Smart? Students' Positioning in High School Mathematics. In V. Hand (Chair), *Identity and Mathematics Learning,* Interactive Paper Session to be conducted at the annual meeting of the National Council of Teachers of Mathematics (NCTM) Research Pre-session, Indianapolis, IN.


APPENDIX A

Patterns and Categories of Positioning and Participation in Social Interactions

*1. Format of engagement in relation to the task
i = In
o = Out

Forms of disengagement (out)
ph = Physical
v = Verbal
b = Both

*2. Area of engagement
*S = Social (how a student matters as a person as they work, quality of social interaction)
*M = Mathematics (how students’ mathematics participation evolves)
L = Language

Types of Participation (any of these qualifies either S or M from previous variable)
Pv = Privileged (central account of one’s action & participation over others, given externally by someone)
C = Central: as expert (certain, knowing what to do) or novice (interested, unsure, learning what to do)
D = Dominant (enacting own will and imposing one’s participation without accounting for others’)
P = Peripheral (simply observing attentive, passive mode seems optional, decided by student)
m = Marginal (overtly put down/excluded by group)
N = Ignored (no attention whatsoever to student case)
2 = Secondary (active participation, but others’ claims, Actions, ideas count stronger)
Si = Silenced (in participation, but communication process stopped, shut down)
Otr = Othered (Singling out, creating differences between student and the group)
n = Null (Student is not ignored, nor has a central participation, does not care to be in or out; she/he is there but not significant to him/her or others)

Forms of Social Process
C = Collaborative (elaborating on each other’s actions)
I = Individual (working on their own)

Modes input:
Sio- OR Sio+ = Singling out (could be either positive –praising, show as an example—or negative—bring out publically errors mistakes without the purpose of helping)
r = Repair (errors are pointed and worked through)
f = Forced (task imposed on student)

Modes of output
Mb = Misbehavior (acting out overtly, not complying, not doing whatever was expected)
F = Fight back (talk back aggressively validating one’s point or actions)
p = Persistence (continue on doing the same despite limitations, diligence)
R = Resistance (refusal to give in, but not aggressive)
**Didactic Situation:**
t = Tutor-like (explain step by step how to do something to another)
d = Discussion (converse about what is being done)
g = Game (apply rules, play a game that is not a task or project)

*/Language*
Eng = English (English only or used mostly with few embedded words or phrases in Sp)
Sp = Spanish (Spanish only or used mostly with few embedded words or phrases in Eng)
Sp & Eng = Both, almost equal amounts
Eng < Sp = English with little Spanish, Spanish is noticeable, but not in equal amounts
Sp < Eng = Spanish with little English, English is noticeable, but not in equal amounts
SpEn = Spanglish (This one will be marked whenever happens at least once)

**Types of Positions:**
PF = Powerful
PE = Equalized
PL = Powerless
PN = Null
IX. VITA

Carlos Alfonso López Leiva
Ph. D. in Curriculum and Instruction (Curriculum Studies–Mathematics Education)
Email: clopez3@uic.edu

EDUCATION & CERTIFICATIONS

Dissertation: Positioning and Latinas/os: A Study of Small-Group Interactions in Mathematics. Longitudinal, qualitative, case study of the co-construction of the positioning and opportunities of four students during their participation in mathematics problem solving in small-group interactions. It describes how positioning is a recurrent, yet flexible process where students strategically use codeswitching as a process to position each other and gain mathematical authority during problem solving.


1993-2001 B.A., Educational Psychology, Universidad Rafael Landívar, Campus Central, Guatemala City, Guatemala
Thesis: The IQ and its influence in the effectiveness of the Harvard-Venezuela Intelligence Development Program. Study developed during the BA in Educational Psychology Program at Universidad Rafael Landívar in Guatemala City (2001). It explored the modifiability of middle school students’ intelligence through an intervention program on observation developed by Harvard University.

1988-1993 Secondary Education Certification Program, Universidad Rafael Landívar, Quetzaltenango, Guatemala
Study: Spelling development of the teacher candidates in the Secondary Education Program at the Universidad Rafael Landívar in Quetzaltenango. Work required and collectively developed during the Secondary Education Program, Universidad Rafael Landívar in Quetzaltenango, Guatemala (1991)

1985-1987 Elementary School Education Certification, Colegio Evangélico “La Patria”, Quetzaltenango, Guatemala

AREAS OF RESEARCH AND TEACHING INTEREST

- Teacher development for multicultural classrooms
- Discourse & social interactions classrooms
- Learning difficulties (struggling learners)
- Mathematics teaching & learning
- Bilingualism & intercultural education
- Critical pedagogy
- Parental involvement
- Out-of-school learning
RESEARCH EXPERIENCE
Research Assistant Fellowship in the Center for the Mathematics Education of Latinos/as (CEMELA); developing various studies on bilingualism and mathematics teaching and learning, and equity at The University of Illinois at Chicago, Chicago, IL (2006-present).

RESEARCH GRANTS

PUBLICATIONS

Manuscripts in Preparation

Other Manuscripts

PRESENTATIONS


López Leiva, C. A. (2010, October). Juxtaposing mathematical identities: Same students with different contexts, perspectives, and languages. Presented at the Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA), Columbus, OH.

López Leiva, C. A., & Velázquez, G. (2010, October). Emergence of language, power, and authority relations in group work. Presented at the annual meeting of the Society for Advancement of Chicanos and Native Americans in Science (SACNAS), Anaheim, CA.


López Leiva, C. (2009, October). Power structures infiltrating into the zone of proximal development: Uncovering exclusionary practices. In R. Gutiérrez (Chair), Math gaps and pipelines: Does addressing equity mean something more in mathematics education? Symposium conducted at the annual meeting of the Society for Advancement of Chicanos and Native Americans in Science (SACNAS), Dallas, TX.


López Leiva, C. A. (1999). Lessons learned developing an integrated content area, student-centered approach that promotes reading and writing skills in first grade students. Presented at the Teacher School Council meeting in Quetzaltenango, Guatemala.

PROFESSIONAL EXPERIENCE:

University Level Teaching
University of Illinois at Chicago, Chicago, IL (2003-to present)

Adjunct Instructor in the Department of Curriculum and Instruction, and Special Education in the College of Education. Teaching various courses:

Undergraduate Courses:
ED 342 Teaching and Learning Mathematics in Elementary School (Fall 2009, Fall 2010)

Combined Undergraduate/Graduate Courses:
CI 482 Assessment & Instruction: A Multilingual/multicultural Perspective (Summer, 2010)
SPED 473 Teaching Math and Science with Adaptations (Spring 2008)

Graduate Courses
CI 507 Teaching and Learning Mathematics in Elementary School (Spring, 2010)
Research Assistant in the Center for the Mathematics Education of Latinos/as (CEMELA) NSF funded Center. Principal Investigators: Marta Civil, University of Arizona; Lena Licón Khisty, UIC. Center committed to exploring the linguistic, cultural, and social resources of the mathematics teaching and learning of Latino/a students. My function was coordinating the Afterschool Program and developing related research projects with the research team (Fall 2006-Fall 2010).

Assistant Coordinator in Project FLAME (Family Literacy Aprendiendo, Mejorando, y Enseñando Program) from UIC, assisting coordination of Project FLAME and processing payroll monthly (Fall 2004-Spring 2006).

Teaching Assistant & Literacy Instructor in Project FLAME from UIC, Teaching ESL and coordinating workshops for Latino parents to develop their children’s literacy skills in their native language at home (Fall 2003-Fall 2005).

Universidad Rafael Landívar, Quetzaltenango, Guatemala (2003).

Instructor in the College of Education in the Secondary Education Program, teaching General Pedagogy and Learning Disabilities courses at the undergraduate level in two sections during the first year of High School Teacher Education Program (2003, during weekends).

K-12 Level Teaching

Telpochalli Elementary School, Chicago, IL (2005-2006)
Special Education Assistant, working in 1st-8th grades bilingual Dual Language Program assisting special education teachers in different subject areas (Mathematics, Science, Language Arts) working with students in small groups (2005-2006).

Elementary School Principal in charge of educational affairs coordinating and supervising teachers´ planning, school curricula, and school activities; and developing support and PD training sessions on Current Educational Reform Topics for staff teachers (1997-2003).

Elementary School Teacher, teaching different grades and positions, three years as an English teacher in all Elementary School grades; one year in 6th grade and five years in 1st teaching all subjects; three years teaching Spanish and Social Studies in 4th, 5th, and 6th grades (1988-2001).

Middle School Teacher, teaching courses such as Mathematics, Accounting, Logic, and Social Studies in grades 7th-9th (1994-1998, during afternoons).


Other Professional Experiences

Inglés en Español, Language School, Quetzaltenango, Guatemala (2002-3).
English Instructor, teaching English to students ages 4-8 at the beginning level using the Total Physical Responsive Approach and Whole Language perspectives, meeting three times a week in afternoons (7/2002-1/2003).
The Pan American Social Marketing Organization (PASMO), Quetgo, Guatemala (2002).

*Educational Advisor*, coordinating an HIV Prevention Program, working with communities at risk of infection such as Sexual Workers, Gay people, Truck drivers, men wearing uniforms and people living with HIV. (1/2002-12/2002)


*Teacher of Spanish as a Foreign Language* during vacation time from elementary school, between October-January. Teaching Spanish one-on-one to international students of different ages (10/1997 to 01/2000).

*Assistant Coordinator*, during Internship at a Community Development Project of “La Pedrera” at Casa de Español Xelajú, Quetzaltenango’s Suburbs; working with women, and children in the community on projects such as: Sewing, Ceramics, Sexual Education, Cooking, Art Crafts, Christmas crafts, and applying rehabilitation programs to three children of the community with Learning disabilities (1995).

**PROFESSIONAL SERVICE**

University of Illinois at Chicago, *Critical Race Studies in Education Association*, Chicago IL

*Assistant Coordinator*, member of the Critical Race Studies in Education Association organizing team, assisting in the logistical arrangements for the Annual Conferences in Tucson, AZ and Salt Lake City, UT (2009-to present).

University of Illinois at Chicago & *Carpenter Elementary School*, Chicago IL

*Academic Support Consultant*, collaborating with a Chicago Public School teacher (Mr. John Hillier) in discussing, observing, planning, and debriefing about mathematics teaching strategies, task selection, and language and cultural identity issues in a 5th grade multicultural classroom having a majority of bilingual Latino/a students. At times, this process also evolved into co-teaching situations. Meeting took place at school during planning time and observation the rest of the day, once a week during school year (Fall 2008-Spring 2009).

University of Illinois at Chicago & *“Los Rayos” Math Club Afterschool Program*, Chicago IL

*Mentor*, in collaboration with graduate students, I worked with a group of pre-service teachers in a mathematics afterschool promoting reflective situations that allowed us to think and discuss issues of language, identity, and culture related to mathematics and bilingual education (Fall 2007-Spring 2008).

University of Illinois at Chicago, Chicago IL

*Consultant*, I collaborated with a student, Brenda Sandoval, in the psychology department working on her community project developed at “El Valor” a Latino community center in Chicago for bilingual families and students. Her project focused on developing interventions in the early childhood program for two bilingual students with disabilities (Fall 2008).

Universidad Rafael Landívar, Quetzaltenango, Guatemala.

*Planning Team member*, coordinating with other faculty members the logistics and academics of a symposium: Effective teaching strategies for pre-service teachers in the Secondary Education Program.
Guatemala Intercultural Travel Agency, Quetzaltenango, Guatemala.

Child Sponsorship Program Coordinator working at the Guatemala Intercultural Travel Agency in Quetzaltenango. Looking for sponsoring Godparents for low SES children’s education and school supplies and maintaining communication with both sides (2003, weekdays afternoons).

Educational Department of Quetzaltenango, Quetzaltenango, Guatemala, (1995-97)
Member of Board of School Counselors Community in Quetzaltenango; planning, organizing, and carrying out workshops for middle and High school counselors in Quetzaltenango, Guatemala (1995-96).

HIV prevention Educator, preparing activities and workshops and facilitating Holistic Workshop Programs to at-risk Minorities in Quetzaltenango working with OASIS, an Organization Supporting a Congruent Attitude toward Aids and Minorities (1997).

Member of the Counseling Group of HIV Prevention at Quetzaltenango, Guatemala. Planning and organizing related activities (1997).

ACADEMIC HONORS AND AWARDS

Research Fellowship in the Center for the Mathematics Education of Latinos/as (CEMELA) at The University of Illinois at Chicago (UIC), Chicago, IL (2006-present).

Dean’s Merit Award, after completion of M.Ed. in Special Education, College of Education University of Illinois at Chicago, Chicago, Illinois (2006).

Teacher of the Year Award, State Educational Office in Quetzaltenango, Guatemala, (2001).

President of the leading research committee during the study Spelling development of the teacher candidates in the Secondary Education Program at the Universidad Rafael Landívar in Quetzaltenango. Work required and collectively developed during the Secondary Education Program, Universidad Rafael Landívar in Quetzaltenango, Guatemala (1991).

Junior Year Abroad Scholarship, granted by the United States Agency for International Development (USAID) and Government of Guatemala to study in the College of Education, University of West Florida, Pensacola, Florida (1989-1990).

President of the leading research committee during the development of the monograph: The Society’s influence on child development. Collectively developed at the Elementary School Education Program at Colegio Evangélico “La Patria”, Quetzaltenango, Guatemala (1987).

Valedictorian Award, graduating from Elementary Education Program at the Colegio Evangélico “La Patria”, Quetzaltenango, Guatemala (1987).